

## On the macroscopic parameters of brittle fracture

N.F. MOROZOV and Y.V. PETROV (ST. PETERSBURG)

THE PROBLEM of modeling of some of the specific effects of brittle fracture in the high loading rate conditions is discussed. An approach based on the system of fixed material constants describing macro-strength properties of the material is considered. New principles of material testing are analyzed. The corresponding incubation time criterion allows us to manage without the *a priori* given rate dependences of dynamic strength and fracture toughness. New applications of the criterion to the problems of disintegration and erosion are considered.

### 1. Basic structural characteristic of static fracture

ONE OF THE PRINCIPAL parameters of linear fracture mechanics is the material structure size  $d$  describing the elementary cell of failure. The classical approaches by GRIFFITH [3] and IRWIN [5] consider this characteristic as a latent quantity. It may be presented as dimensional combination of surface energy, critical stress intensity factor, static strength and elastic constants of the material:

$$(1.1) \quad d \sim \frac{\gamma \cdot E}{\sigma_c^2}, \quad d \sim \frac{K_{Ic}^2}{\sigma_c^2}.$$

The elementary cell of fracture has no unique physical interpretation. It may be interpreted in various ways, depending on the class of problems. The corresponding Griffith – Irwin criterion is a universally recognized critical condition of brittle and quasi-brittle fracture. This criterion is based on the square root singularity of the stress field at the crack tip. Therefore its field of applicability is limited. For instance, in the case of an angular notch in a plate, the general energy balance equation cannot be satisfied for all methods of loading (MOROZOV [9]).

NEUBER [12] and NOVOZHILOV [14] suggested to consider the material structure directly. The corresponding criterion requires that the mean normal stress in the range of material structure size  $d$  must be equal to the static strength of the material. In the plane deformation state case we have:

$$(1.2) \quad \frac{1}{d} \int_0^d \sigma(r) dr \leq \sigma_c.$$

Assuming that in the simplest cases the criterion (1.2) gives the same results as the Irwin's critical stress intensity factor criterion, we obtain for the material structure size  $d$  the expression:

$$(1.3) \quad d = \frac{2}{\pi} \frac{K_{Ic}^2}{\sigma_c^2}.$$

Criterion (1.2) may be used in various cases in which the square root singularity and the appropriate energy balance do not work. Results obtained by means of the criterion (1.2) under the condition (1.3) are well confirmed by experiments in static cases (MOROZOV [9]).

The introduced material constant  $d$  is quite similar to the process zone size parameter for the short crack fracture assessment occurring in ceramics (ANDO *et al.* [1]).

## 2. Fast fracture processes

The constant progress observed in experimental mechanics during the last years enables us to understand the fact that dynamic fracture in brittle solids is remarkable for its specific nature. The corresponding experiments helped us to discover some principal effects that have no interpretation within the framework of the conventional models of brittle failure. Here are just some of them:

1) The dynamic branch of the time-dependence of strength and fracture delays in spalling (ZLATIN *et al.* [23]).

2) The extensive zones of failure (cavitation) in spalling and their unpredictable geometry (BROBERG [2], SEAMAN *et al.* [21], etc.).

3) The dependence of critical stress intensity factor of crack growth initiation on the loading history (RAVI-CHANDAR and KNAUSS [20]).

4) The behaviour of the short pulse load threshold amplitudes leading to failure at the crack tip (KALTHOFF and SHOCKEY [6], SHOCKEY *et al.* [22]).

Analysis of the experiments shows that the main contradictions of the traditional models appear when failure occurs during the short time intervals after the start of the loading process. MOROZOV and PETROV [10, 11] proposed an approach to the analysis of dynamic brittle failure based on the incubation time criterion:

$$(2.1) \quad \frac{1}{\tau} \int_{t-\tau}^t \frac{1}{d} \int_0^d \sigma(r, \theta, t') dr dt' \leq \sigma_c.$$

Here  $d$  and  $\tau$  are material structure size and structure time of failure, respectively,  $\sigma_c$  is static strength of the material,  $(r, \theta)$  are polar coordinates,  $\sigma(r, \theta, t)$  is tensile stress at the crack tip ( $r = 0$ ). The material structure size  $d$  is to be determined in accordance with the data of quasi-static tests of specimens containing cracks, and in the case of plane strain it may be expressed by the simple formula (1.3). The material structure time  $\tau$  is responsible for the dynamic peculiarities of the macro-fracture process and for each material it should be found from experiments. In accordance with this approach,  $\sigma_c$ ,  $K_{Ic}$  and  $\tau$  constitute the system of fixed material constants describing macro-strength properties of the material. PETROV [15] has shown that the criterion (2.1) reflects the discrete nature of dynamic fracture of brittle solids.

In the case of virgin materials, the criterion (2.1) reduces to the form:

$$(2.2) \quad \frac{1}{\tau} \int_{t-\tau}^t \sigma(t') dt' \leq \sigma_c.$$

This form will now be used for the analysis of two particular problems.

The analysis of the particular problems of dynamic fracture mechanics is associated with the appropriate choice of the parameter  $\tau$ . We shall mention two basic cases:

1. The incubation time is defined by the material structure size of fracture:

$$(2.3) \quad \tau = \frac{d}{c} = \frac{d\sqrt{\rho}}{k},$$

where  $c$  is the maximum wave velocity,  $\rho$  is the density of continuum,  $k$  is the constant depending on the deformation material properties. According to this definition, the incubation time has a physical meaning of the minimum time period required for the interaction between two neighbouring material structure cells. The incubation-time criterion with the parameter  $\tau$  selected according to the formula (2.3) allows us to describe effectively the time-dependence of strength and the fracture zone geometry in conditions of spalling (MOROZOV *et al.* [10, 11], PETROV and UTKIN [16]). Thus, the definition (2.3) provides a good analogy between the incubation time criterion and the well-known experiments in the case of “defectless” materials.

2. The incubation time does not directly depend on the material structure size of failure. This takes place when a problem of initiation of the macro-crack growth is considered. Nucleation, growth and coalescence of micro-defects in the special process zone region at the crack tip precedes the growth of the macro-crack. These processes are accompanied by a local stress relaxation and change the effective material properties. The incubation time  $\tau$  is to be considered as the principal integral characteristic of the processes in the corresponding process zone region. PETROV and MOROZOV [18] proved that in the case of macro-cracks, the material structure time  $\tau$  can be interpreted as an incubation time in the well-known minimum time criterion proposed and explored by KALTHOFF and SHOCKEY [6], HOMMA *et al.* [4], and SHOCKEY *et al.* [22]:

$$(2.4) \quad \tau = t_{\text{inc}}.$$

The aforementioned dependence of the fracture toughness on the loading history and the specific behaviour of the short loading pulse threshold amplitudes can be explained and effectively analyzed by means of the incubation time criterion under the condition (2.4) (PETROV and MOROZOV [18]).

### 3. Some basic principles of the material strength properties testing

In this section we outline some of the possible methods of description of the material strength properties. Table 1 represents the basic parameters and criteria to be used in testing of the materials. In Table 1  $\sigma_c$ ,  $K_{Ic}$  are the material constants,  $\sigma_c^{\text{dyn}}(v)$ ,  $K_{I_d}(v)$  are the material functions that represent the dependences of critical characteristics on the loading rate  $v$ .

Table 1.

No.	Method	Material parameters	Criteria
1	Classical static	$\sigma_c, K_{Ic}$	$\sigma \leq \sigma_c, K_I \leq K_{Ic}$
2	Classical dynamic	$\sigma_c^{\text{dyn}}(v), K_{I_d}(v)$	$\sigma(t) \leq \sigma_c^{\text{dyn}}, K_I(t) \leq K_{I_d}$
3	SRI International	$\sigma_c^{\text{dyn}}(v), K_{I_d}(v), t_{\text{inc}}$	$\sigma(t) \leq \sigma_c^{\text{dyn}}$ , minimum time criterion
4	Incubation time approach	$\sigma_c, K_{Ic}, \tau$	incubation time criterion

The classical dynamic approach, resulting directly from the static strength theory and linear fracture mechanics, is based on two strength characteristics  $\sigma_c^{\text{dyn}}(v)$ ,  $K_{I_d}(v)$ , that are supposed to be material functions found from experiments.

The minimum time theory proposed by J.F. Kalthoff, D.A. Shockey and co-workers is based on the incubation time notion. It allows us to explain some of the principal dynamic fracture effects. On the other hand, the minimum time technique turns out to be too sophisticated for practical engineering.

It is seen from the Table 1 and the aforementioned results that the incubation time criterion combines the simplicity of the classical static method with the effectiveness of the SRI International approach. Basing on the system of fixed material constants, it enables us to predict the behaviour of dynamic strength and dynamic fracture toughness from a unified viewpoint. Thus, the rate strength dependences may be considered as calculated characteristics. The criterion may be applied for both the "defectless" and macro-cracked specimens.

### 4. Application to the problem of disintegration

The great interest in connection with the technology of disintegration of solids is presented by the issues concerning the speed of propagation of failure and the fracture zone geometry. Even elementary experiments on spalling show that the zone of destruction may have very diverse geometry. It can be either one spalling section (like a crack), or several ones. In some experiments, the zone of fracture has a form of a continuously damaged domain of finite extension. This domain was named a zone of continuous crushing (NIKIFOROVSKY and SHEMJAKIN [13]). There are also experiments, in which the zone of destruction represents the mixture

of damaged regions with intact parts of the material. Most of the experiments show, that the geometry of the failure zones strongly depends on the parameters of the applied loading, such as a speed of loading, its amplitude, duration, etc. Eventually, it can be said that the whole history of loading is very important.

In this section we consider the particular problem of disintegration of a solid ball caused by the instant discharge (unloading) of the external pressure (PETROV, SEMENOV and UTKIN [17]). We shall demonstrate that the incubation time criterion gives all the variety of the fracture zones just in the sonic approximation of the problem.

Let a ball be loaded on its surface by a uniform pressure. At a certain moment, the pressure is instantaneously taken away. The corresponding boundary value problem in the sonic approximation is described by the following system of equations:

$$(4.1) \quad \begin{aligned} u &= u(r, t), & u &= \frac{\partial \Phi}{\partial r}, & \sigma &= -\frac{\partial \Phi}{\partial t}, \\ \frac{\partial^2 \Phi}{\partial t^2} &= c^2 \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right), \\ -\frac{\partial \Phi}{\partial t} \Big|_{r=R} &= [1 - H(t)] \cdot \sigma_0. \end{aligned}$$

Here  $H(t)$  is the Heaviside step function;  $u$  and  $\sigma$  are the displacement and pressure inside the material,  $\Phi$  is the potential function,  $\sigma_0$  is the initial pressure,  $c$  is the wave velocity.

The discharge of the pressure produces a spherical wave of unloading moving to the center of the ball. This wave carries a tensile stress that produces fracture of the material. The aforementioned incubation time criterion allows us to find the extent and geometry of failure in this particular situation.

We assume that the material is homogeneous, isotropic and composed of the spherical layers of thickness  $b$ . All layers are assumed to have identical material properties. We assume that the layer is destroyed at the certain moment, when the critical condition in the center of the layer corresponding to the criterion (2.2) is fulfilled. The destroyed layer turns out to be, on the one hand, a shield for the moving waves, and on the other hand, a source of an additional unloading wave. The unloading waves running inside the ball are reflected from the destroyed layers and interact with other stress waves. Thus the process is characterized by a complicated picture of direct and reflected waves.

The scheme was realized in a complex of computer programs BALL. One of the main results of the calculations was the graphically submitted zone of fracture. It turned out that the region of failure strongly depends on the parameters of the problem and on the material constants. Some of the possible variants of the fracture zone for the hypothetical values  $\sigma_c = 900$  MPa,  $\tau = 1 \mu\text{s}$ ,  $b = 3$  mm,  $c = 5000$  m/s are presented in the Fig.1 (fracture zones are shown in black), where a large variety of geometry is seen.

Strength=900MPa; Inc.time=1mcs; b=3mm; c=5000m/s

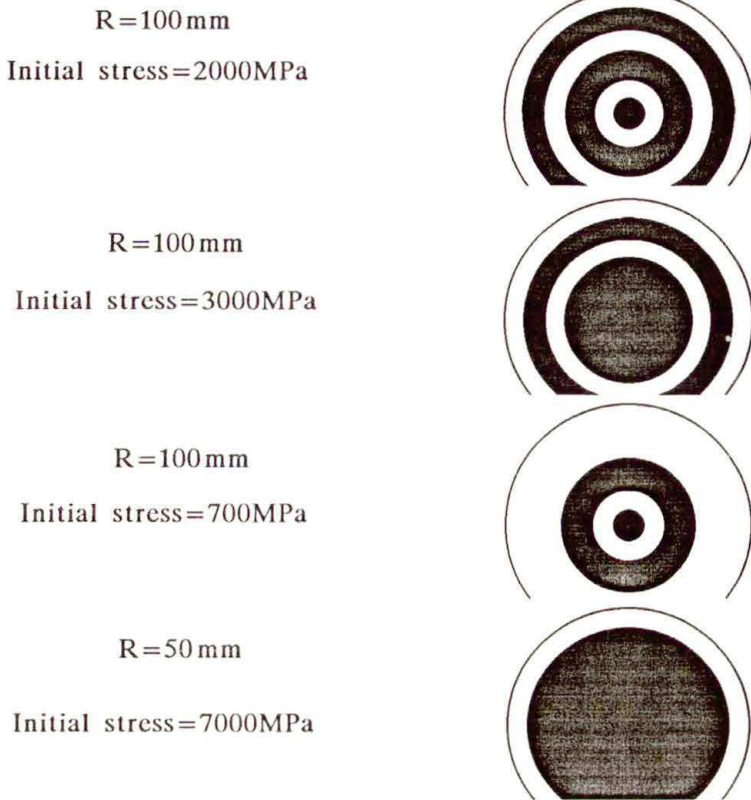


FIG. 1. Calculated fracture zones caused by instant unloading of the ball.

## 5. Application to the problem of erosion

The solid particle impact velocity at the beginning of target material loss in the steady-state erosion process can be considered as a critical or threshold velocity. It is a principal characteristic that bears an information about dynamic strength properties of materials subjected to the impact loading. In this section, the relation between the threshold velocity  $W$  and the incubation time  $\tau$  is investigated. The possibility of using the incubation time criterion in determining the threshold erosion characteristics is established.

One of the principal features of the erosion process is that the target material surface is subjected to extremely short impact actions. The evaluation of failure in these conditions may be done only on the basis of special criteria reflecting the specific nature of fast fracture phenomenon. The incubation time criterion (2.2) is an effective instrument for this analysis. Here we shall consider the simplest way to obtain some of the basic threshold erosion characteristics.

Let a spherical particle of radius  $R$  fall with velocity  $v$  on the surface of an elastic half-space. Using the classical Hertz impact theory approximation (KOLESNIKOV and MOROZOV [7]), we describe the motion of the particle by the following equation:

$$(5.1) \quad m \frac{d^2 h}{dt^2} = -P,$$

where

$$(5.2) \quad P(t) = k(R)h^{3/2}(t), \quad k(R) = \frac{4}{3}\sqrt{R} \frac{E}{(1-\nu^2)}.$$

At the beginning of the impact event we have  $dh/dt = v$ . The maximum penetration  $h_0$  occurs when  $dh/dt = 0$ . Solving Eq. (5.1), we obtain

$$(5.3) \quad h_0(v, R) = \left( \frac{5mv^2}{4k} \right)^{2/5}, \quad t_0(v, R) = \frac{2h_0}{v} \int_0^1 \frac{d\gamma}{\sqrt{1-\gamma^{5/2}}} = 2.94 \frac{h_0}{v},$$

where  $t_0$  is the duration of the impact event. The penetration function  $h(t)$  can be approximated by the simple formula (KOLESNIKOV and MOROZOV [7]):

$$(5.4) \quad h(t) = h_0 \sin(\pi t/t_0).$$

The maximum tensile stress occurring at the edge of the contact area is given by the expression (LAWN and WILSHAW [8]):

$$(5.5) \quad \sigma(t, v, R) = \frac{1-2\nu}{2} \frac{P(t, v, R)}{\pi a^2(t, v, R)},$$

$$a(t, v, R) = \left[ 3P(t, v, R)(1-\nu^2) \frac{R}{4E} \right]^{1/3},$$

where the contact force  $P(t, v, R)$  can be found by means of Eqs. (5.2)–(5.4).

Let  $v = W$  denote the threshold velocity corresponding to the beginning of failure. We consider the function:

$$(5.6) \quad f(\tau, v, R) = \max_t \int_{t-\tau}^t \sigma(s, v, R) ds - \sigma_c \tau.$$

According to (2.2), we determine the threshold velocity  $v = W$  as the minimum positive root of the equation:

$$(5.7) \quad f(\tau, v, R) = 0,$$

where  $\tau$  is the incubation time for the target material.

The corresponding calculations were performed for the aluminum alloy B95 and the incubation time was determined according to formulae (1.3), (2.3):  $\sigma_c = 460$  MPa,  $K_{Ic} = 37$  MPa m<sup>1/2</sup>,  $c = 6500$  m/s,  $\tau = 2K_{Ic}^2/(\pi\sigma_c^2c) \approx 0.6$   $\mu$ s. The calculated dependence of the threshold velocity  $W$  on the value of radius  $R$  is presented in the Fig. 2 by the solid curve. The static branch shows a weak dependence of the threshold velocity on the length of the radius. On the contrary, the dynamic branch, corresponding to the small particles and very short loading pulses, represents a strong dependence of the critical velocity on the radius of particles. This behaviour of the threshold velocity is observed in numerous experiments (POLEZHAEV [19]), but it can not be explained on the basis of the traditional fracture mechanics. The dependence following from the conventional critical stress theory is also presented in the Fig. 2 by dashed line.

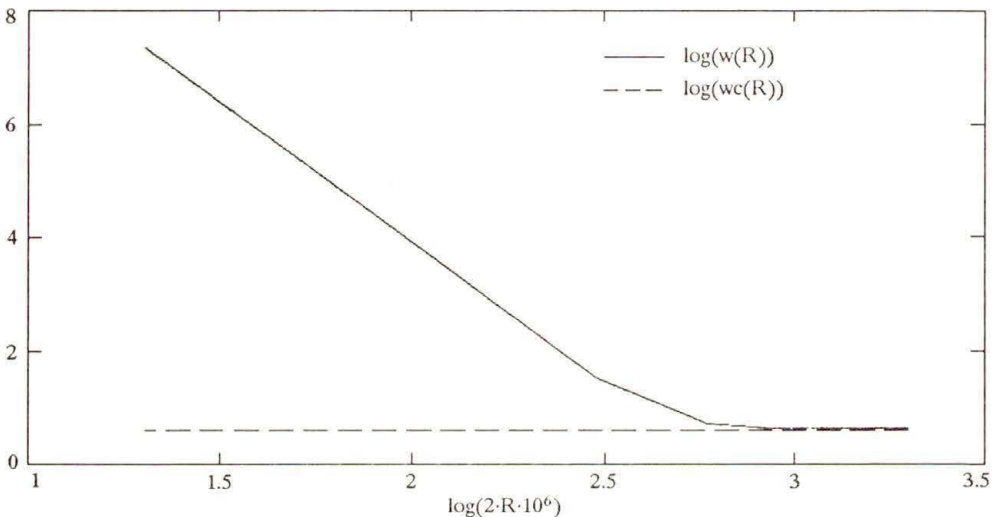


FIG. 2. Dependence of the threshold velocity  $W$  (m/s) on the radius  $R$  (m) of erodent particles calculated for aluminium alloy B95. The dependence corresponding to the classical fracture criterion:  $\sigma \leq \sigma_c$  is plotted by dashed line.

## Acknowledgments

The research described in this publication was made possible owing partly to the Grants 96-01-00391 and 95-01-00644 from the Russian Fundamental Research Foundation (RFFI).

## References

1. K. ANDO, B.A. KIM, M. IWASA and N. OGURA, *Process zone size failure criterion and probabilistic fracture assessment curves for ceramics*, *Fatigue Fract. Engng. Mater. Struct.*, 15, 2, pp. 139–149, 1992.
2. K.B. BROBERG, *Some aspects of the mechanism of scabbing*, [in:] *Stress Wave Propagate Material*, N.-Y.-L., Interscience, pp. 229–246, 1960.



3. A. GRIFFITH, *The phenomena of rupture and flow in solids*, Philos. Trans. Roy. Soc., London, Ser. A **221**, 1921.
4. H. HOMMA, D.A. SHOCKEY, and Y. MURAYAMA, *Response of cracks in structural materials to short pulse loads*, J. Mech. Phys. Solids, **31**, pp. 261–279, 1983.
5. G. IRWIN, *Analysis of stresses and strains near the end of a crack traversing a plate*, J. Appl. Mech., **3**, 1957.
6. J.F. KALTHOFF and D.A. SHOCKEY, *Instability of cracks under impulse loads*, J. Appl. Physics, **48**, pp. 986–993, 1977.
7. Y.V. KOLESNIKOV and E.M. MOROZOV, *Contact fracture mechanics* [in Russian], Nauka, Moscow 1989.
8. B.R. LAWN and T.R. WILSHAW, *Indentation fracture: principles and application*, J. Mater. Sci., **10**, 6, pp. 1049–1081, 1975.
9. N.F. MOROZOV, *Mathematical questions of crack theory* [in Russian], Nauka, Moscow 1990.
10. N.F. MOROZOV and Y.V. PETROV, *Dynamic fracture toughness in crack growth initiation problems* [in Russian], Izv. AN SSSR. MTT (Solid Mechanics), **6**, pp. 108–111, 1990.
11. N.F. MOROZOV, Y.V. PETROV and A.A. UTKIN, *On the analysis of spalling in the framework of structural fracture mechanics* [in Russian], Dokl. AN SSSR, **313**, 2, pp. 276–279, 1990.
12. H. NEUBER, *Kerbspannungslehre (Notch Stresses)*, Julius Verlag, Berlin 1937.
13. V.S. NIKIFOROVSKY and E.I. SHEMIKIN, *Dynamic fracture of solids* [in Russian], Nauka, Novosibirsk 1979.
14. V.V. NOVOZHILOV, *Necessary and sufficient criterion of brittle strength* [in Russian], Prikl. Matematika i Mechanika, **33**, 2, pp. 212–222, 1969.
15. Y.V. PETROV, *On “quantum” nature of dynamic fracture in brittle media* [in Russian], Dokl. AN SSSR, **321**, 1, pp. 66–68, 1991.
16. Y.V. PETROV and A.A. UTKIN, *Structure-time peculiarities of dynamic fracture of materials*, [in:] Macro- and Micro-Mechanical Aspects of Fracture, Abstracts, Proc. of the EUROMECH-291, St. Petersburg, Russia, June 22-27, 1992.
17. Y.V. PETROV, B.N. SEMENOV and A.A. UTKIN, *Dynamic fragmentation of solids caused by the instant discharge of intensive pressure* [in Russian], Proc. of the 1st Russian Symp. on Solid Mechanics, St. Petersburg, June 6-9, 1994.
18. Y.V. PETROV and N.F. MOROZOV, *On the modeling of fracture of brittle solids*, ASME J. Applied Mech., **61**, pp. 710–712, 1994.
19. Y.V. POLEZHAEV, *Thermo-gas-dynamic testing of aircrafts* [in Russian], Izd. MAI, Moscow 1986.
20. K. RAVI-CHANDAR and W.G. KNAUSS, *An experimental investigation into dynamic fracture. I. Crack initiation and arrest*, Int. J. Fract., **25**, pp. 247–262, 1984.
21. L. SEAMAN, D.R. CURRAN and W.J. MURRI, *A continuum model for dynamic tensile microfracture and fragmentation*, J. Appl. Mech., **52**, pp. 593–600, 1985.
22. D.A. SHOCKEY, D.C. ERLICH, J.F. KALTHOFF and H. HOMMA, *Short pulse fracture mechanics*, Engng. Fract. Mech., **23**, pp. 311–319, 1986.
23. N.A. ZLATIN, G.S. PUGAVHEV, S.M. MOCHALOV and A.M. BRAGOV, *Time-dependence of strength of metals* [in Russian], FTT, **17**, 9, pp. 2599–2602, 1975.

ACADEMY OF SCIENCES OF RUSSIA  
STRENGTH AND FRACTURE MECHANICS GROUP  
ST. PETERSBURG, RUSSIA.

Received December 22, 1995.