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Reflection and refraction of plane waves at the interface of an elastic solid half-space and a thermoelastic diffusive solid half-space

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The problem of reflection and refraction phenomenon due to longitudinal and transverse waves incident obliquely at a plane interface between uniform elastic solid half-space and thermoelastic diffusive solid half-space has been studied. It is found that the amplitude ratios of various reflected and refracted waves are functions of angle of incidence, frequency of incident wave and are influenced by the elastic properties of media. The expressions of amplitude ratios and energy ratios are obtained in closed form. The amplitude ratios and energy ratios have been computed numerically for a particular model. The variations of energy ratios with angle of incidence are shown graphically. The conservation of energy across the interface is verified. Some particular cases are also discussed.

Key words: thermoelastic diffusive solid, elastic waves, reflection, refraction, amplitude and energy ratios.

1. Introduction

In classical theory of thermoelasticity, Fourier's heat conduction theory assumes that the thermal disturbances propagate at infinite speed which is unrealistic from the physical point of view. Two different generalizations of the classical theory of thermoelasticity have been developed which predict only finite velocity of propagation for heat and displacement fields. The first one is given by Lord and Shulman [18] which incorporates a flux rate term into the Fourier's law of heat conduction and formulates a generalized theory admitting finite speed for thermal signals. The second is given by Green and Lindsay [14] which developes a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables, which does not violate the classical Fourier's law of heat conduction. Lord and Shulman [18] theory of generalized thermoelasticity have been further extended to homogeneous anisotropic heat conducting materials recommended by Dhaliwal and Sherief [9]. All these theories predict a finite speed of heat propagation. Chanderashekhariah [8] refers to this wave-like thermal disturbance as second sound. A survey article of various representative theories in the range of generalized thermoelasticity have been brought out by Hetnarski and Ignaczak [15].

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g., xexon) and other light isotopes (e.g., carbon) for research purposes. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of temperature on this interaction. However, there is a certain degree of coupling with temperature and temperature gradients as temperature speeds up the diffusion process. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with the environment.

Nowacki [19–22] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Dudziak and Kowalski [10] and Olesiak and Pyryev [23], respectively, discussed the theory of thermodiffusion and coupled quasi-stationary problems of thermal diffusion for an elastic layer. They studied the influence of cross-effects arising from the coupling of the fields of temperature, mass diffusion and strain due to which the thermal excitation results in additional mass concentration and that generates additional fields of temperature. Gawinecki *et al.* [12] proved a theorem about existence, uniqueness and regularity of the solution to an initial-boundary value problem for a nonlinear coupled parabolic system. They used an energy method, method of Sobolev spaces, semigroup theory and Banach fixed point theorem to prove the theorem. Gawinecki and Szymaniec [13] proved a theorem about global existence of the solution to the initial-value problem for a nonlinear hyperbolic parabolic system of coupled partial differential equation of second order describing the process of thermodiffusion in solid body. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief *et al.* [25] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Due to the inherit complexity of the derivation of the variational principle equations, Aouadi [4] proved this theorem in the Laplace transform domain, under the assumption that the functions of the problem are continuous and the inverse Laplace transform of each is also unique. Sherief and Saleh [26] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Kumar and Kansal [16] developed the basic equation of anisotropic thermoelastic diffusion based upon Green–Lindsay model.

Abd-alla and Alsheikh [1] studied a problem of reflection and refraction of quasi-longitudinal waves under initial stresses at an interface of two anisotropic piezoelectric media with different properties. Abd-alla *et al.* [2] discussed propagation of plane vertical transverse waves at an interface of a semi-infinite piezoelectric elastic medium under the influence of the initial stresses. Borejko [6] discussed the reflection and transmission coefficients for three-dimensional plane waves in elastic media. Wu and Lundberg [29] investigated the problem of reflection and transmission of the energy of harmonic elastic waves in a bent bar. Sinha and Elsibai [27] discussed the reflection and refraction of thermoelastic waves at an interface of two semi-infinite media with two relaxation times. Sharma and Gogna [24] discussed the problem of reflection and refraction of plane harmonic waves at an interface between elastic solid and porous solid saturated by viscous liquid. Tomar and Arora [28] studied reflection and transmission of elastic waves at an elastic/porous solid saturated by immiscible fluids. Kumar and Sarthi [17] discussed the reflection and refraction of thermoelastic plane waves at an interface of two thermoelastic media without energy dissipation.

In the present paper, the reflection and refraction phenomenon at a plane interface between an elastic solid medium and a thermoelastic diffusive solid medium has been analyzed. In thermoelastic diffusive solid medium, potential functions are introduced to represent three longitudinal waves and one transverse wave. The amplitude ratios of various reflected and refracted waves to that of incident wave are derived. These amplitude ratios are further used to find the expressions of energy ratios of various reflected and refracted waves to that of incident wave. The graphical representation is given for these energy ratios for different direction of propagation. The law of conservation of energy at the interface is verified.

2. Basic equations

Following Sherief *et al.* [25] and Kumar and Kansal [16], the basic equations of homogeneous isotropic generalized thermoelastic diffusive solid in the absence of body forces, heat and mass diffusive sources are:

(i) constitutive relations:

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau^1\dot{C})], \tag{2.1}$$

$$\rho T_0 S = k + \rho C_E(T + \alpha\dot{T}) + \beta_1 T_0 e_{kk} + aT_0(C + \beta\dot{C}), \tag{2.2}$$

$$P = -\beta_2 e_{kk} + b(C + \tau^1\dot{C}) - a(T + \tau_1\dot{T}); \tag{2.3}$$

(ii) equations of motion:

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(T + \tau_1\dot{T})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho\ddot{u}_i; \tag{2.4}$$

(iii) equation of heat conduction:

$$\rho C_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0(\dot{e}_{kk} + \varepsilon\tau_0\ddot{e}_{kk}) + aT_0(\dot{C} + \gamma\ddot{C}) = KT_{,ii}; \tag{2.5}$$

(iv) equation of mass diffusion:

$$D\beta_2 e_{kk,ii} + Da(T + \tau_1\dot{T})_{,ii} + (\dot{C} + \varepsilon\tau^0\ddot{C}) - Db(C + \tau^1\dot{C})_{,ii} = 0, \tag{2.6}$$

where $\beta_1 = (3\lambda+2\mu)\alpha_t$ and $\beta_2 = (3\lambda+2\mu)\alpha_c$; λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion and α_c is the coefficient of linear diffusion expansion. a, b are, respectively, coefficients describing the measure of thermodiffusion and of mass diffusion effects, $T = \Theta - T_0$ is small temperature increment; Θ is the absolute temperature of the medium; T_0 is the reference temperature of the body chosen such that $|T/T_0| \ll 1$, C is the concentration of the diffusive material in the elastic body. u_i are the components of the displacement vector $\mathbf{u}$, ρ is the density assumed to be independent of the time, σ_{ij}, e_{ij} $(= \frac{1}{2}(u_{i,j} + u_{j,i}))$ are the components of the stress and strain tensors respectively, e_{kk} is the dilatation, S is the entropy per unit mass, P is the chemical potential per unit mass, C_E is the specific heat at the constant strain, K is the coefficient of the thermal conductivity, D is the thermoelastic diffusion constant, k is a material constant. τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $\alpha = \beta = k = \tau_1 = \tau^1 = 0$, $\varepsilon = 1$, $\gamma = \tau_0$ for Lord–Shulman (L-S) model and $\alpha = \tau_0$, $\beta = \tau^0$, $\varepsilon = 0$, $\gamma = \tau^0$ for Green–Lindsay (G-L) model. In the above equations, a comma followed by a suffix denotes spatial derivative and a superposed dot denotes the derivative with respect to time.

The basic equations of homogeneous isotropic elastic solid are written as:

$$\mu^e u^e_{i,jj} + (\lambda^e + \mu^e)u^e_{j,ij} = \rho^e\ddot{u}^e_i, \tag{2.7}$$

where λ^e, μ^e are Lame's constants, u^e_i are the components of the displacement vector $\mathbf{u}^e$, ρ^e is density corresponding to isotropic elastic solid.

The stress tensor σ^e_{ij} in the isotropic elastic solid medium is given by:

$$\sigma^e_{ij} = 2\mu^e e^e_{ij} + \lambda^e e^e_{kk}\delta_{ij}, \tag{2.8}$$

where e^e_{ij} $(= \frac{1}{2}(u^e_{i,j} + u^e_{j,i}))$ are the components of the strain tensor, e^e_{kk} is the dilatation.

3. Formulation of the problem

We consider an isotropic elastic solid half-space lying over a homogeneous isotropic, generalized thermoelastic diffusive solid half-space. The origin of the Cartesian coordinate system (x_1, x_2, x_3) is taken at any point on the plane surface(interface) and x_3-axis points vertically downwards into the thermoelastic diffusive solid half-space. The elastic solid half-space occupies the region $x_3 \leq 0$ (medium I) and the region $x_3 \geq 0$ is occupied by the dissipative thermoelastic diffusive solid half-space (medium II) as shown in Fig. 1. We consider plane waves in the x_1–x_3 plane with wave front parallel to the x_2-axis. For two-dimensional problem, the displacement vectors $\mathbf{u}^e$ in medium I and $\mathbf{u}$ in medium II are taken as:

$$\mathbf{u}^e = (u^e_1, 0, u^e_3), \qquad \mathbf{u} = (u_1, 0, u_3). \tag{3.1}$$

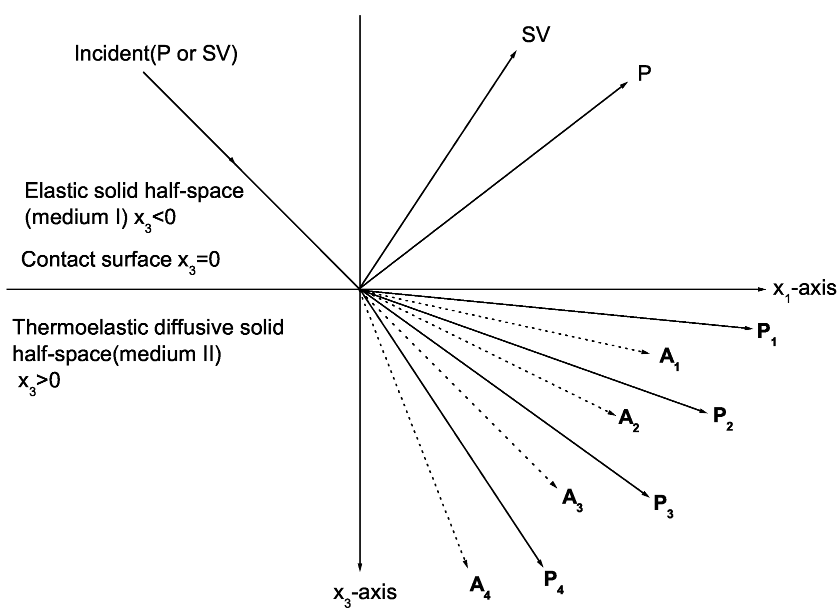

Fig. 1. Geometry of the problem.

We define the following dimensionless quantities:

$$
\begin{aligned}
&x_1' = \frac{w_1^* x_1}{c_1}, \quad x_3' = \frac{w_1^* x_3}{c_1}, \quad u_1' = \frac{w_1^* u_1}{c_1}, \quad u_3' = \frac{w_1^* u_3}{c_1},\\
&u_1^{e'} = \frac{w_1^* u_1^e}{c_1}, \quad u_3^{e'} = \frac{w_1^* u_3^e}{c_1}, \quad t' = w_1^* t,\\
&T' = \frac{\beta_1 T}{\rho c_1^2}, \quad C' = \frac{\beta_2 C}{\rho c_1^2},\\
&\tau_0' = w_1^* \tau_0, \quad \tau_1' = w_1^* \tau_1, \quad \tau^{0'} = w_1^* \tau^0, \quad \tau^{1'} = w_1^* \tau^1,\\
&\sigma_{ij}' = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad \sigma_{ij}^{e'} = \frac{\sigma_{ij}^e}{\beta_1 T_0}, \quad P_{ij}^{*'} = \frac{P_{ij}^*}{\beta_1 T_0 c_1}, \quad P^{*e'} = \frac{P^{*e}}{\beta_1 T_0 c_1},
\end{aligned}
\tag{3.2}
$$

where $w_1^* = \rho C_E c_1^2/K$, $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$.

Upon introducing the quantities (3.2) in Eqs. (2.4)–(2.6) with the aid of (3.1) and after suppressing the primes, we obtain:

$$
(1-\delta^2)[u_{1,11} + u_{3,13}] + \delta^2[u_{1,11} + u_{1,33}] - \tau_t^1 T_{,1} - \tau_c^1 C_{,1} = \ddot{u}_1, \tag{3.3}
$$

$$
(1-\delta^2)[u_{1,13} + u_{3,33}] + \delta^2[u_{3,11} + u_{3,33}] - \tau_t^1 T_{,3} - \tau_c^1 C_{,3} = \ddot{u}_3, \tag{3.4}
$$

$$
T_{,11} + T_{,33} = \tau_t^0 \dot{T} + \zeta_1 \tau_c^0 \dot{C} + \zeta_2 \tau_e^0 [\dot{u}_{1,1} + \dot{u}_{3,3}], \tag{3.5}
$$

$$
\begin{aligned}
q_1^*[u_{1,111} + u_{1,133} + u_{3,111} + u_{3,333}]&\\
+ q_2^* \tau_t^1 [T_{,11} + T_{,33}] - q_3^* \tau_c^1 [C_{,11} + C_{,33}] + \tau_f^0 \dot{C} &= 0,
\end{aligned}
\tag{3.6}
$$

where

$$
\begin{aligned}
&c_2 = \sqrt{\frac{\mu}{\rho}}, \quad \delta^2 = \frac{c_2^2}{c_1^2},\\
&\zeta_1 = \frac{a T_0 c_1^2 \beta_1}{w_1^* K \beta_2}, \quad \zeta_2 = \frac{\beta_1^2 T_0}{\rho K w_1^*},\\
&q_1^* = \frac{D w_1^* \beta_2^2}{\rho c_1^4}, \quad q_2^* = \frac{D w_1^* \beta_2 a}{\beta_1 c_1^2}, \quad q_3^* = \frac{D w_1^* b}{c_1^2},\\
&\tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \quad \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t},\\
&\tau_c^0 = 1 + \gamma \frac{\partial}{\partial t}, \quad \tau_e^0 = 1 + \varepsilon \tau_0 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \varepsilon \tau^0 \frac{\partial}{\partial t}.
\end{aligned}
$$

We introduce the potential functions ϕ and ψ through the relations:

$$
u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \qquad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}, \tag{3.7}
$$

where ϕ and ψ are the displacement potentials of longitudinal and transverse waves.

Substituting equation (3.7) in Eqs. (3.3)–(3.6), we obtain:

$$\nabla^2\phi - \tau_t^1 T - \tau_c^1 C = \ddot{\phi}, \tag{3.8}$$

$$\nabla^2\psi - \frac{\ddot{\psi}}{\delta^2} = 0, \tag{3.9}$$

$$\nabla^2 T = \tau_t^0 \dot{T} + \zeta_1 \tau_c^0 \dot{C} + \zeta_2 \tau_e^0 \nabla^2 \dot{\phi}, \tag{3.10}$$

$$q_1^* \nabla^4 \phi + q_2^* \tau_t^1 \nabla^2 T - q_3^* \tau_c^1 \nabla^2 C + \tau_f^0 \dot{C} = 0, \tag{3.11}$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.$$

Assuming the motion to be harmonic, we can write:

$$\{\phi, \psi, T, C\}(x_1, x_3, t) = \{\bar{\phi}, \bar{\psi}, \bar{T}, \bar{C}\} e^{-\iota\omega t}, \tag{3.12}$$

where ω is the angular frequency of vibrations of material particles. Substituting the expressions of ϕ, ψ, T, C into Eqs. (3.8)–(3.11), we obtain:

$$[\nabla^2 + \omega^2]\bar{\phi} - \tau_t^{11}\bar{T} - \tau_c^{11}\bar{C} = 0, \tag{3.13}$$

$$\left[\nabla^2 + \frac{\omega^2}{\delta^2}\right]\bar{\psi} = 0, \tag{3.14}$$

$$-\zeta_2 \tau_e^{10} \nabla^2 \bar{\phi} + [\nabla^2 - \tau_t^{10}]\bar{T} - \zeta_1 \tau_c^{10} \bar{C} = 0, \tag{3.15}$$

$$q_1^* \nabla^4 \bar{\phi} + q_2^* \tau_t^{11} \nabla^2 \bar{T} - [q_3^* \tau_c^{11} \nabla^2 - \tau_f^{10}]\bar{C} = 0, \tag{3.16}$$

where

$$\tau_t^{11} = 1 - \iota\omega\tau_1, \qquad \tau_c^{11} = 1 - \iota\omega\tau^1, \qquad \tau_t^{10} = -\iota\omega(1 - \iota\omega\tau_0),$$
$$\tau_c^{10} = -\iota\omega(1 - \iota\omega\gamma), \qquad \tau_e^{10} = -\iota\omega(1 - \iota\omega\varepsilon\tau_0), \qquad \tau_f^{10} = -\iota\omega(1 - \iota\omega\varepsilon\tau^0).$$

Equations (3.15) and (3.16) of this system are solved into two relations, given by:

$$\begin{aligned}\{[q_3^* \tau_c^{11} \zeta_2 \tau_e^{10} + q_1^* \zeta_1 \tau_c^{10}]\nabla^4 - \zeta_2 \tau_e^{10} \tau_f^{10} \nabla^2\}\bar{\phi} \\ = \{q_3^* \tau_c^{11} \nabla^4 - [q_3^* \tau_c^{11} \tau_t^{10} + \tau_f^{10} + q_2^* \tau_t^{11} \zeta_1 \tau_c^{10}]\nabla^2 + \tau_t^{10} \tau_f^{10}\}\bar{T},\end{aligned} \tag{3.17}$$

$$\begin{aligned}\{q_1^* \nabla^6 + [q_2^* \tau_t^{11} \zeta_2 \tau_e^{10} - q_1^* \tau_t^{10}]\nabla^4\}\bar{\phi} \\ = \{q_3^* \tau_c^{11} \nabla^4 - [q_3^* \tau_c^{11} \tau_t^{10} + \tau_f^{10} + q_2^* \tau_t^{11} \zeta_1 \tau_c^{10}]\nabla^2 + \tau_t^{10} \tau_f^{10}\}\bar{C}.\end{aligned} \tag{3.18}$$

Using relations (3.17) and (3.18) in Eq. (3.13), we obtain:

$$[G_1\nabla^6 + G_2\nabla^4 + G_3\nabla^2 + G_4]\bar{\phi} = 0, \tag{3.19}$$

where

$$\begin{aligned}
G_1 &= (q_1^* - q_3^*)\tau_c^{11},\\
G_2 &= \tau_f^{10} + (q_1^* + q_2^*)\zeta_1\tau_c^{10}\tau_t^{11} + (q_2^* + q_3^*)\zeta_2\tau_t^{11}\tau_e^{10}\tau_c^{11}\\
&\quad - (q_1^* - q_3^*)\tau_t^{10}\tau_c^{11} - q_3^*\tau_c^{11}\omega^2,\\
G_3 &= \tau_f^{10}(\omega^2 - \tau_t^{10} - \zeta_2\tau_t^{11}\tau_e^{10}) + q_3^*\tau_t^{10}\tau_c^{11}\omega^2 + q_2^*\zeta_1\tau_c^{10}\tau_t^{11}\omega^2,\\
G_4 &= -\omega^2\tau_f^{10}\tau_t^{10}.
\end{aligned}$$

The general solution of Eq. (3.19) can be written as:

$$\bar{\phi} = \bar{\phi}_1 + \bar{\phi}_2 + \bar{\phi}_3, \tag{3.20}$$

where the potentials $\bar{\phi}_i$, $i = 1, 2, 3$ are solutions of wave equations, given by:

$$\left[\nabla^2 + \frac{\omega^2}{V_i^2}\right]\bar{\phi}_i = 0, \qquad i = 1, 2, 3. \tag{3.21}$$

Here V_1, V_2 and V_3 are the velocities of three longitudinal waves, that is, P, MD (Mass Diffusive) and T (Thermal) waves and derived from the roots of cubic equations in V^2, given by:

$$G_4V^6 - G_3\omega^2V^4 + G_2\omega^4V^2 - G_1\omega^6 = 0. \tag{3.22}$$

From Eq. (3.14), we obtain:

$$[\nabla^2 + \frac{\omega^2}{V_4^2}]\bar{\psi} = 0, \tag{3.23}$$

where $V_4 = \delta$ is the velocity of transverse wave.

Making use of Eq. (3.20) in Eqs. (3.17) and (3.18) with the aid of Eqs. (3.12) and (3.21), the general solutions for ϕ, T and C are obtained as:

$$\{\phi, T, C\} = \sum_{i=1}^{3}\{1, n_i, k_i\}\phi_i, \tag{3.24}$$

where

$$n_i = \frac{[q_3^*\tau_c^{11}\zeta_2\tau_e^{10} + q_1^*\zeta_1\tau_c^{10}]\omega^4 + \zeta_2\tau_e^{10}\tau_f^{10}\omega^2V_i^2}{\tau_t^{10}\tau_f^{10}V_i^4 + [q_3^*\tau_c^{11}\tau_t^{10} + \tau_f^{10} + q_2^*\tau_t^{11}\zeta_1\tau_c^{10}]\omega^2V_i^2 + q_3^*\tau_c^{11}\omega^4},$$

$$k_i = \frac{-q_1^*\omega^6 + [q_2^*\tau_t^{11}\zeta_2\tau_e^{10} - q_1^*\tau_t^{10}]\omega^4 V_i^2}{V_i^2[\tau_t^{10}\tau_f^{10}V_i^4 + [q_3^*\tau_c^{11}\tau_t^{10} + \tau_f^{10} + q_2^*\tau_t^{11}\zeta_1\tau_c^{10}]\omega^2 V_i^2 + q_3^*\tau_c^{11}\omega^4]}, \quad i = 1,2,3.$$

Applying the dimensionless quantities (3.2) in Eq. (2.7) with the aid of (3.1) and after suppressing the primes, we obtain:

$$\frac{(\alpha^{e^2} - \beta^{e^2})}{c_1^2}[u^e_{1,11} + u^e_{3,13}] + \frac{\beta^{e^2}}{c_1^2}[u^e_{1,11} + u^e_{1,33}] = \ddot{u}^e_1, \tag{3.25}$$

$$\frac{(\alpha^{e^2} - \beta^{e^2})}{c_1^2}[u^e_{1,13} + u^e_{3,33}] + \frac{\beta^{e^2}}{c_1^2}[u^e_{3,11} + u^e_{3,33}] = \ddot{u}^e_3, \tag{3.26}$$

where $\alpha^e = \sqrt{(\lambda^e + 2\mu^e)/\rho^e}$, $\beta^e = \sqrt{\mu^e/\rho^e}$ are velocities of longitudinal and transverse waves corresponding to medium I, respectively.

The components u_1^e and u_3^e are related by the potential functions as:

$$u_1^e = \frac{\partial\phi^e}{\partial x_1} - \frac{\partial\psi^e}{\partial x_3}, \qquad u_3^e = \frac{\partial\phi^e}{\partial x_3} + \frac{\partial\psi^e}{\partial x_1}, \tag{3.27}$$

where ϕ^e and ψ^e satisfy the wave equations as:

$$\nabla^2\phi^e = \frac{\ddot{\phi}^e}{\alpha'^2}, \qquad \nabla^2\psi^e = \frac{\ddot{\psi}^e}{\beta'^2}, \tag{3.28}$$

where $\alpha' = \alpha^e/c_1$ and $\beta' = \beta^e/c_1$.

4. Reflection and refraction

We consider a harmonic wave (P or SV) propagating through the isotropic elastic solid half-space and is incident at the interface $x_3 = 0$ as shown in Fig. 1. Corresponding to this incident wave, two homogeneous waves (P and SV) are reflected in isotropic elastic solid half-space and four inhomogeneous waves (P, MD, T and SV) are refracted in isotropic thermoelastic diffusive solid half-space.

In elastic solid half-space, the potential functions satisfying Eq. (3.28) can be written as

$$\phi^e = A_0^e e^{[\iota\omega\{(x_1\sin\theta_0 + x_3\cos\theta_0)/\alpha' - t\}]} + A_1^e e^{[\iota\omega\{(x_1\sin\theta_1 - x_3\cos\theta_1)/\alpha' - t\}]}, \tag{4.1}$$

$$\psi^e = B_0^e e^{[\iota\omega\{(x_1\sin\theta_0 + x_3\cos\theta_0)/\beta' - t\}]} + B_1^e e^{[\iota\omega\{(x_1\sin\theta_2 - x_3\cos\theta_2)/\beta' - t\}]}. \tag{4.2}$$

The coefficients A_0^e, B_0^e, A_1^e and B_1^e represent the amplitudes of the incident P (or SV), reflected P and reflected SV-waves, respectively.

Following BORCHERDT [5], in isotropic thermoelastic diffusive solid half-space, the potential functions satisfying Eqs. (3.21) and (3.23) can be written as:

$$\{\phi, T, C\} = \sum_{i=1}^{3} \{1, n_i, k_i\} B_i e^{(\mathbf{A}_i \cdot \mathbf{r})} e^{\{\iota(\mathbf{P}_i \cdot \mathbf{r} - \omega t)\}}, \tag{4.3}$$

$$\psi = B_4 e^{(\mathbf{A}_4 \cdot \mathbf{r})} e^{\{\iota(\mathbf{P}_4 \cdot \mathbf{r} - \omega t)\}}. \tag{4.4}$$

The coefficients B_i, $i = 1, 2, 3, 4$ represent the amplitudes of refracted P, MD, T and SV-waves, respectively. The propagation vector $\mathbf{P}_i$, $i = 1, 2, 3, 4$ and attenuation factor $\mathbf{A}_i$, $i = 1, 2, 3, 4$ are given by:

$$\mathbf{P}_i = \xi_R \hat{x}_1 + dV_{iR} \hat{x}_3, \qquad \mathbf{A}_i = -\xi_I \hat{x}_1 - dV_{iI} \hat{x}_3, \quad i = 1, 2, 3, 4, \tag{4.5}$$

where

$$dV_i = dV_{iR} + \iota dV_{iI} = \text{p.v.}\left(\frac{\omega^2}{V_i^2} - \xi^2\right)^{1/2}, \; i = 1, 2, 3, 4. \tag{4.6}$$

and $\xi = \xi_R + \iota \xi_I$ is a complex wave number. The subscripts R and I denote the real and imaginary parts of the corresponding complex quantity and p.v. stands for the principal value of the complex quantity obtained after square root. $\xi_R \geq 0$ ensures propagation in the positive x_1-direction. The complex wave number ξ in the isotropic thermodiffusive elastic solid medium is given by:

$$\xi = |\mathbf{P}_i| \sin\theta_i' - \iota\, |\mathbf{A}_i| \sin(\theta_i' - \gamma_i), \quad i = 1, 2, 3, 4, \tag{4.7}$$

where γ_i, $i = 1, 2, 3, 4$ is the angle between the propagation and attenuation vector and θ_i', $i = 1, 2, 3, 4$ is the angle of refraction in medium II.

5. Boundary conditions

The boundary conditions to be satisfied at the interface $x_3 = 0$ are:
(i) continuity of stress components:

$$\sigma_{33}^e = \sigma_{33}, \tag{5.1}$$

$$\sigma_{31}^e = \sigma_{31}; \tag{5.2}$$

(ii) continuity of displacement components:

$$u_1^e = u_1, \tag{5.3}$$

$$u_3^e = u_3; \tag{5.4}$$

(iii) thermally insulated boundary:

$$\frac{\partial T}{\partial x_3} = 0; \tag{5.5}$$

(iv) impermeable boundary:

$$\frac{\partial C}{\partial x_3} = 0. \tag{5.6}$$

Making the use of potentials given by Eqs. (4.1)–(4.4), we find that the boundary conditions are satisfied if and only if:

$$\xi_R = \frac{\omega \sin \theta_0}{V_0} = \frac{\omega \sin \theta_1}{\alpha'} = \frac{\omega \sin \theta_2}{\beta'}, \tag{5.7}$$

and

$$\xi_I = 0, \tag{5.8}$$

where

$$V_0 = \begin{cases} \alpha', & \text{for incident P-wave,} \\ \beta', & \text{for incident SV-wave.} \end{cases} \tag{5.9}$$

It means that waves are attenuating only in x_3-direction. From Eq. (4.7), it implies that if $|\mathbf{A}_i| \neq 0$, then $\gamma_i = \theta_i'$, $i = 1, 2, 3, 4$, that is, attenuated vectors for the four refracted waves are directed along the x_3-axis.

Using Eqs. (4.1)–(4.4) in the boundary conditions (5.1)–(5.6) with the aid of Eqs. (3.7), (3.27), (5.7)–(5.9), we get a system of six non-homogeneous equations which can be written as:

$$\sum_{j=1}^{6} d_{ij} Z_j = g_i, \tag{5.10}$$

where $Z_j = |Z_j| e^{\iota \psi_j^*}$, $|Z_j|$, ψ_j^*, $j = 1, \ldots, 6$ represent amplitudes ratios and phase shift of reflected P-, reflected SV-, refracted P-, refracted MD-, refracted T- and refracted SV-waves to that of incident wave, respectively.

$$d_{11} = 2\mu^e \left(\frac{\xi_R}{\omega}\right)^2 - \rho^e c_1^2, \qquad d_{12} = 2\mu^e \frac{\xi_R}{\omega} \frac{dV_{\beta'}}{\omega},$$

$$d_{16} = 2\mu \frac{\xi_R}{\omega} \frac{dV_4}{\omega}, \qquad d_{21} = 2\mu^e \frac{\xi_R}{\omega} \frac{dV_{\alpha'}}{\omega},$$

$$d_{22} = \mu^e \left[\left(\frac{dV_{\beta'}}{\omega}\right)^2 - \left(\frac{\xi_R}{\omega}\right)^2\right], \qquad d_{26} = \mu \left[\left(\frac{\xi_R}{\omega}\right)^2 - \left(\frac{dV_4}{\omega}\right)^2\right],$$

$$d_{31} = \frac{\xi_R}{\omega}, \qquad d_{32} = \frac{dV_{\beta'}}{\omega}, \qquad d_{36} = \frac{dV_4}{\omega},$$
$$d_{41} = -\frac{dV_{\alpha'}}{\omega}, \qquad d_{42} = \frac{\xi_R}{\omega}, \qquad d_{46} = -\frac{\xi_R}{\omega},$$
$$d_{51} = d_{52} = d_{56} = 0, \qquad d_{61} = d_{62} = d_{66} = 0,$$
$$d_{1j} = \lambda\left(\frac{\xi_R}{\omega}\right)^2 + \rho c_1^2\left(\frac{dV_j}{\omega}\right)^2 + \frac{\rho c_1^2(n_j\tau_t^{11} + k_j\tau_c^{11})}{\omega^2}, \qquad d_{2j} = 2\mu\frac{\xi_R}{\omega}\frac{dV_j}{\omega},$$
$$d_{3j} = -\frac{\xi_R}{\omega}, \qquad d_{4j} = -\frac{dV_j}{\omega}, \qquad d_{5j} = n_j\frac{dV_j}{\omega}, \qquad d_{6j} = k_j\frac{dV_j}{\omega}, \quad j = 3,4,5,$$
$$\frac{dV_{\alpha'}}{\omega} = \left(\frac{1}{\alpha'^2} - \left(\frac{\xi_R}{\omega}\right)^2\right)^{1/2} = \left(\frac{1}{\alpha'^2} - \frac{\sin^2\theta_0}{V_0^2}\right)^{1/2}, \qquad \frac{dV_{\beta'}}{\omega} = \left(\frac{1}{\beta'^2} - \frac{\sin^2\theta_0}{V_0^2}\right)^{1/2},$$

and

$$\frac{dV_j}{\omega} = \text{p.v.}\left(\frac{1}{V_j^2} - \frac{\sin^2\theta_0}{V_0^2}\right)^{1/2}, \quad j = 1,2,3,4.$$

Here p.v. is evaluated with restriction $dV_{jI} \geq 0$ to satisfy decay condition in thermoelastic diffusive medium. The coefficients g_i, $i = 1,2,3,4$ on the right side of the Eq. (5.10) are given by:

(i) for incident P-wave:

$$g_1 = -d_{11}, \quad g_2 = d_{21}, \quad g_3 = -d_{31}, \quad g_4 = d_{41}, \quad g_5 = 0, \quad g_6 = 0; \tag{5.11}$$

(ii) for incident SV-wave:

$$g_1 = d_{12}, \quad g_2 = -d_{22}, \quad g_3 = d_{32}, \quad g_4 = -d_{42}, \quad g_5 = 0, \quad g_6 = 0. \tag{5.12}$$

Now we consider a surface element of unit area at the interface between two media. The purpose is to calculate the partition of energy of the incident wave among the reflected and refracted waves on the both sides of surface. Following Achenbach [3], the energy flux across the surface element, that is, rate at which the energy is communicated per unit area of the surface is represented as:

$$P^* = \sigma_{lm} l_m \dot{u}_l, \tag{5.13}$$

where σ_{lm} is the stress tensor, l_m are the direction cosines of the unit normal $\hat{\mathrm{l}}$ outward to the surface element and $\dot{u}_l$ are the components of the particle velocity.

The time average of P^* over a period, denoted by $\langle P^*\rangle$, represents the average energy transmission per unit surface area per unit time. Thus, on the surface with normal along x_3-direction, the average energy intensities of the waves in the elastic solid half-space are given by:

$$\langle P^{*e}\rangle = \mathrm{Re}\langle\sigma\rangle_{13}^e\,\mathrm{Re}(\dot{u}_1^e) + \mathrm{Re}\langle\sigma\rangle_{33}^e\,\mathrm{Re}(\dot{u}_3^e). \tag{5.14}$$

Following ACHENBACH [3], for any two complex functions f and g, we have:

$$\langle \mathrm{Re}(f).\mathrm{Re}(g)\rangle = \frac{1}{2}\mathrm{Re}(f.\bar{g}). \tag{5.15}$$

The expressions for energy ratios $E_i,\ i = 1, 2$ for the reflected P- and reflected SV are given by:

$$E_i = -\frac{\langle P_i^{*e}\rangle}{\langle P_0^{*e}\rangle}, \quad i = 1, 2, \tag{5.16}$$

where

$$\langle P_1^{*e}\rangle = \frac{\omega^4 \rho^e c_1^2}{\alpha'} \left|Z_1\right|^2 \mathrm{Re}(\cos\theta_1), \qquad \langle P_2^{*e}\rangle = \frac{\omega^4 \rho^e c_1^2}{\beta'} \left|Z_2\right|^2 \mathrm{Re}(\cos\theta_2),$$

and

(i) for incident P-wave:

$$\langle P_0^{*e}\rangle = -\frac{\omega^4 \rho^e c_1^2}{\alpha'} \cos\theta_0; \tag{5.17}$$

(ii) for incident SV-wave:

$$\langle P_0^{*e}\rangle = -\frac{\omega^4 \rho^e c_1^2}{\beta'} \cos\theta_0; \tag{5.18}$$

are the average energy intensities of the reflected P-, reflected SV-, incident P- and incident SV-waves respectively. In Eq. (5.16), negative sign is taken because the direction of reflected waves is opposite to that of incident wave.

For thermoelastic diffusive solid half-space, the average energy intensities of the waves on the surface with normal along x_3-direction, are given by:

$$\langle P_{ij}^{*}\rangle = \mathrm{Re}\langle\sigma\rangle_{13}^{(i)} \mathrm{Re}(\dot{u}_1^{(j)}) + \mathrm{Re}\langle\sigma\rangle_{33}^{(i)} \mathrm{Re}(\dot{u}_3^{(j)}). \tag{5.19}$$

The expressions for energy ratios $E_{ij},\ i, j = 1, 2, 3, 4$ for the refracted P-, refracted MD-, refracted T- and refracted SV-waves are given by:

$$E_{ij} = \frac{\langle P_{ij}^{*}\rangle}{\langle P_0^{*e}\rangle}, \quad i, j = 1, 2, 3, 4, \tag{5.20}$$

where

$$\langle P_{ij}^{*}\rangle = -\omega^4 \mathrm{Re}\Bigg[\Bigg\{2\mu\frac{dV_i}{\omega}\frac{\xi_R}{\omega}\frac{\bar{\xi}_R}{\omega} + \Bigg\{\lambda\left(\frac{\xi_R}{\omega}\right)^2 + \rho c_1^2\left(\frac{dV_i}{\omega}\right)^2 + \frac{\rho c_1^2(n_i\tau_t^{11} + k_i\tau_c^{11})}{\omega^2}\Bigg\}\frac{d\bar{V}_j}{\omega}\Bigg\} Z_{i+2}\bar{Z}_{j+2}\Bigg],$$

$$\langle P^*_{i4}\rangle = -\omega^4 \mathrm{Re}\Big[\Big\{ -2\mu\frac{dV_i}{\omega}\frac{\xi_R}{\omega}\frac{d\bar{V}_4}{\omega} + \Big\{\lambda\Big(\frac{\xi_R}{\omega}\Big)^2 + \rho c_1^2\Big(\frac{dV_i}{\omega}\Big)^2 + \frac{\rho c_1^2(n_i\tau_t^{11}+k_i\tau_c^{11})}{\omega^2}\Big\}\frac{\bar{\xi}_R}{\omega}\Big\} Z_{i+2}\bar{Z}_6\Big],$$

$$\langle P^*_{4j}\rangle = -\omega^4 \mathrm{Re}\Big[\Big\{\mu\Big(\Big(\frac{\xi_R}{\omega}\Big)^2 - \Big(\frac{dV_4}{\omega}\Big)^2\Big)\frac{\bar{\xi}_R}{\omega} + 2\mu\frac{\xi_R}{\omega}\frac{dV_4}{\omega}\frac{d\bar{V}_j}{\omega}\Big\} Z_6\bar{Z}_{j+2}\Big],$$

$$\langle P^*_{44}\rangle = -\omega^4 \mathrm{Re}\Big[\Big\{-\mu\Big(\Big(\frac{\xi_R}{\omega}\Big)^2 - \Big(\frac{dV_4}{\omega}\Big)^2\Big)\frac{d\bar{V}_4}{\omega} + 2\mu\frac{\xi_R}{\omega}\frac{dV_4}{\omega}\frac{\bar{\xi}_R}{\omega}\Big\} Z_6\bar{Z}_6\Big], \qquad i,j = 1,2,3.$$

The diagonal entries of energy matrix E_{ij} in Eq. (5.20) represent the energy ratios of P, MD, T and SV waves, respectively, whereas sum of the non-diagonal entries of E_{ij} gives the share of interaction energy among all the refracted waves in the medium and is given by

$$E_{\mathrm{RR}} = \sum_{i=1}^{4}\Big(\sum_{j=1}^{4} E_{ij} - E_{ii}\Big). \tag{5.21}$$

The energy ratios E_i, $i = 1,2$, diagonal entries and sum of non diagonal entries of energy matrix E_{ij}, that is, E_{11}, E_{22}, E_{33}, E_{44} and E_{RR} yield the conservation of incident energy across the interface, through the relation:

$$E_1 + E_2 + E_{11} + E_{22} + E_{33} + E_{44} + E_{\mathrm{RR}} = 1. \tag{5.22}$$

6. Particular cases

1. In the absence of diffusion effect, that is, if we take $a = \beta_2 = 0$ in the Eqs. (5.10) and (5.20), we obtain the corresponding expressions for amplitude and energy ratios of reflected P-, reflected SV-, refracted P-, refracted T- and refracted SV-waves to that of incident wave. In these expressions the velocities V_1 and V_3 are derived from the roots of quadratic equation in V^2, given by:

$$\tau_t^{10}V^4 + (\omega^2 - \tau_t^{10} - \zeta_2\tau_t^{11}\tau_e^{10})V^2 - \omega^2 = 0, \tag{6.1}$$

and the coupling coefficients n_i, $i = 1,3$ are given as

$$n_i = \frac{\zeta_2\tau_e^{10}\omega^2}{\tau_t^{10}V_i^2 + \omega^2}, \quad i = 1,3.$$

2. Further, in the absence of thermal and diffusion effects, that is, if we take $a = \beta_1 = \beta_2 = 0$ in Eqs. (5.10) and (5.20), we obtain the corresponding expressions for amplitude and energy ratios of reflected P-, reflected SV-, refracted P-, and refracted SV-waves to that of incident wave which are similar to as obtained in Ewing *et al.* [11] by changing dimensionless quantities into physical quantities.

7. Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections and comparing these in the context of various theories of thermoelastic diffusion, we now represent some numerical results for copper material[26], the physical data for which is given below:

$$\begin{aligned}
\lambda &= 7.76 \times 10^{10}\ \mathrm{Kg\,m^{-1}s^{-2}}, & \mu &= 3.86 \times 10^{10}\ \mathrm{Kg\,m^{-1}s^{-2}},\\
T_0 &= 0.293 \times 10^{3}\ \mathrm{K}, & C_E &= 0.3831 \times 10^{3}\ \mathrm{JKg^{-1}K^{-1}},\\
\alpha_t &= 1.78 \times 10^{-5}\ \mathrm{K^{-1}}, & \alpha_c &= 1.98 \times 10^{-4}\ \mathrm{Kg^{-1}m^{3}},\\
a &= 1.2 \times 10^{4}\ \mathrm{m^{2}s^{-2}K^{-1}}, & b &= 9 \times 10^{5}\ \mathrm{Kg^{-1}m^{5}s^{-2}},\\
D &= 0.85 \times 10^{-8}\ \mathrm{Kg\,s\,m^{-3}}, & \rho &= 8.954 \times 10^{3}\ \mathrm{Kg\,m^{-3}},\\
K &= 0.383 \times 10^{3}\ \mathrm{Wm^{-1}K^{-1}}.
\end{aligned}$$

The relaxation times are:

$$\tau_0 = 0.2\ \mathrm{s}, \qquad \tau_1 = 0.9\ \mathrm{s}, \qquad \tau^0 = 0.3\ \mathrm{s}, \qquad \tau^1 = 0.8\,\mathrm{s}.$$

Following Bullen [7], the numerical data of granite in elastic medium is given by:

$$\rho^e = 2.65 \times 10^{3}\ \mathrm{Kg\,m^{-3}}, \qquad \alpha^e = 5.27 \times 10^{3}\ \mathrm{m\,s^{-1}}, \qquad \beta^e = 3.17 \times 10^{3}\mathrm{m\,s^{-1}}.$$

The software Matlab 7.0.4 has been used to determine the values of energy ratios E_i, $i = 1, 2$ and an energy matrix E_{ij}, $i, j = 1, 2, 3, 4$ defined in the previous section for different values of incident angle (θ_0) ranging from 0° to 90° for fixed frequency $\omega = 2 \times \pi \times 100$ Hz. Corresponding to incident P and SV waves, the variations of these energy ratios with respect to angle of incidence have been plotted in Figs. 2–8 and Figs. 9–15, respectively. In all the figures, the vertical and horizontal lines correspond to L-S(LSD) and G-L(GLD) theories of thermoelastic diffusion.

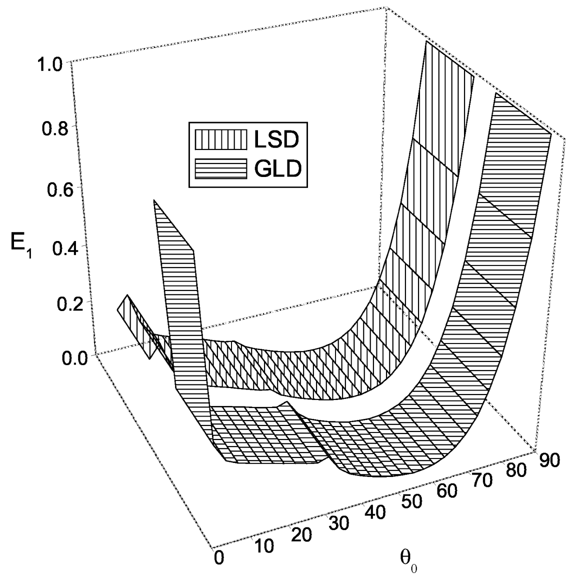

Fig. 2. Variations of energy ratio (E_1) with respect to angle of incidence (θ_0) for P-wave.

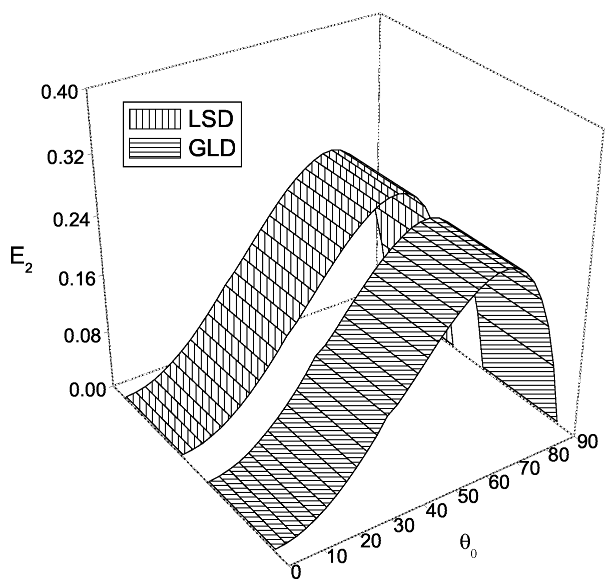

Fig. 3. Variations of energy ratio (E_2) with respect to angle of incidence (θ_0) for P-wave.

Incident P-wave

It is clear from Fig. 2 that for both LSD and GLD theories, the values of energy ratio E_1 decrease with the increase of the angle of incidence (θ_0) from

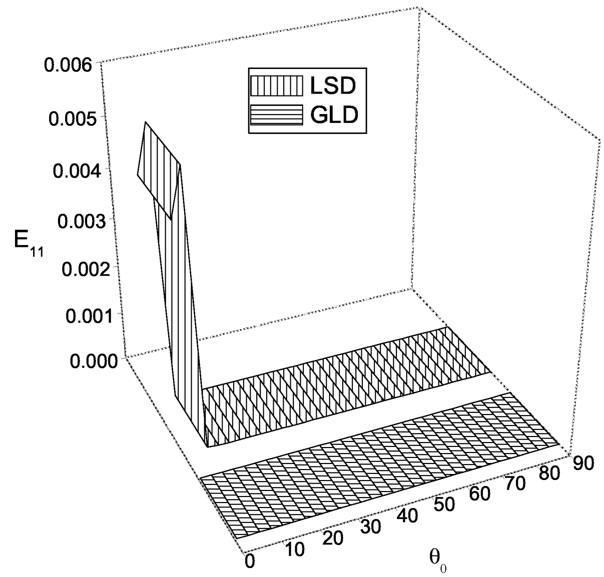

Fig. 4. Variations of energy ratio (E_{11}) with respect to angle of incidence (θ_0) for P-wave.

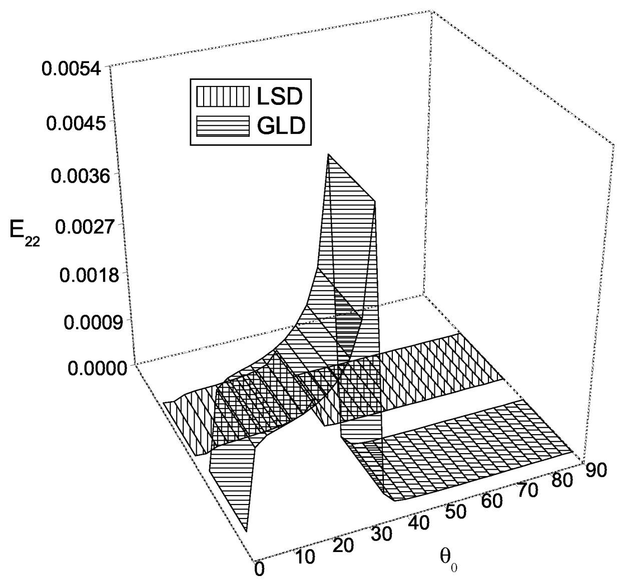

Fig. 5. Variations of energy ratio (E_{22}) with respect to angle of incidence (θ_0) for P-wave.

0° to 63°, and then increase as θ_0 increases further. Figure 3 shows that the values of energy ratio E_2 increase up to at $\theta_0 = 69^\circ$ and thereafter decrease continuously. Figure 4 indicates that for LSD theory, initially for a small range,

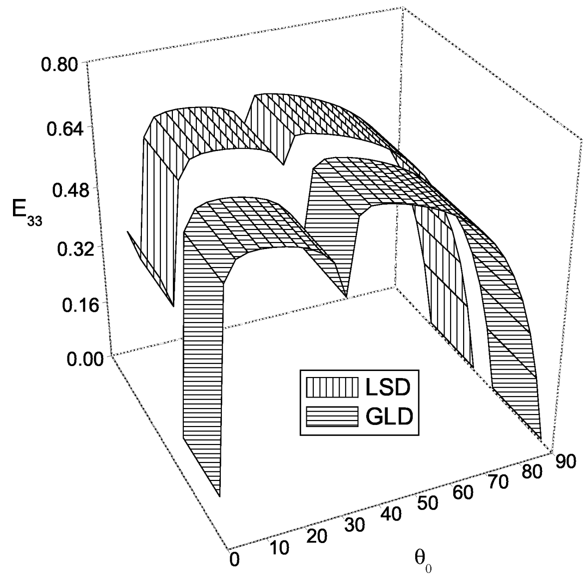

Fig. 6. Variations of energy ratio (E_{33}) with respect to angle of incidence (θ_0) for P-wave.

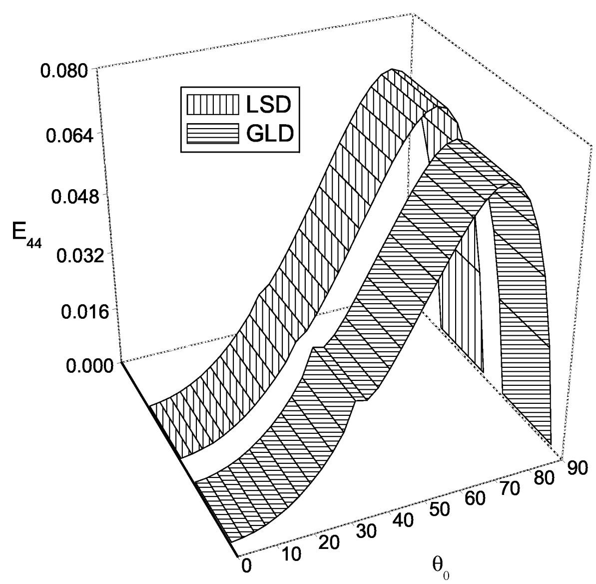

Fig. 7. Variations of energy ratio (E_{44}) with respect to angle of incidence (θ_0) for P-wave.

the value of energy ratio E_{11} increases slightly, but then decreases dramatically. On the other hand, for GLD theory, value of E_{11} decreases for all values of θ_0. Figure 5 depicts that the values of energy ratio E_{22} increase up to half-stage and

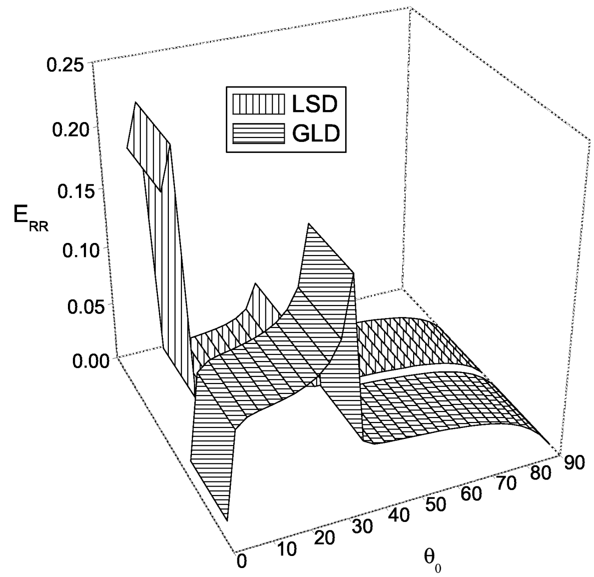

Fig. 8. Variations of energy ratio (E_{44}) with respect to angle of incidence (θ_0) for P-wave.

after that decrease rapidly. From Fig. 6, it is evident that initially, the values of energy ratio E_{33} fluctuate as θ_0 increases, but finally decrease. From Fig. 7, it is noticed that values of energy ratio E_{44} increase to their highest values at $\theta = 72°$ and then decrease continuously. Figure 8 shows that initially, the values of energy ratio E_{RR} show oscillating behavior in the initial stage, but after that decrease slowly and steadily. It is noticed that the sum of the values of energy ratios E_1, E_2, E_{11}, E_{22}, E_{33}, E_{44} and E_{RR} is found to be exactly unity at each value of θ_0 which proves the law of conservation of energy at the interface. However, if we examine Figs. 2–8 closely, we find that sum does not look to be unity. The reason is that we are plotting 3D graphs in origin software. On the other hand, if we plot 2D graphs in any other software, the sum will come out exactly as unity. If we compare two theories in all figures, we find that the values of E_1, E_2, E_{22}, E_{RR} are higher in GLD theory in comparison to LSD theory and the values of E_{11}, E_{33}, E_{44} are more in LSD theory as compared to GLD theory.

Incident SV-wave

From Fig. 9, it is evident that there is a rapid increase in the values of energy ratio E_1 initially, but after $\theta = 36°$ and onwards, values of energy ratio E_1 decrease and become negligible small. Figure 10 depicts that the values of energy ratio E_2 initially fluctuate, but finally reach to nearly unity.

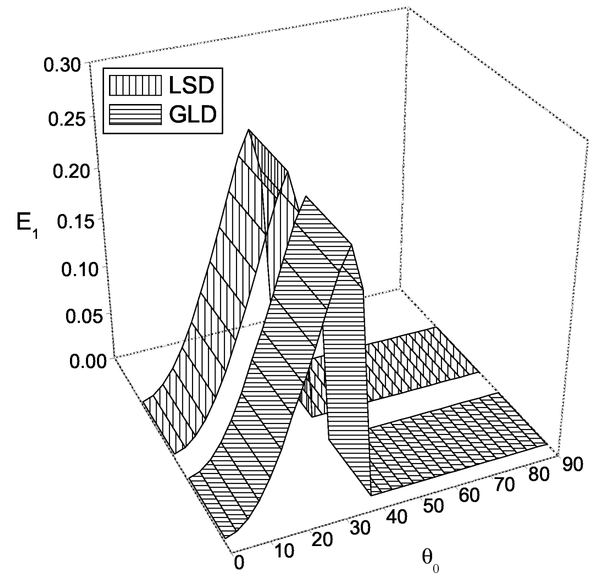

Fig. 9. Variations of energy ratio (E_1) with respect to angle of incidence (θ_0) for SV-wave.

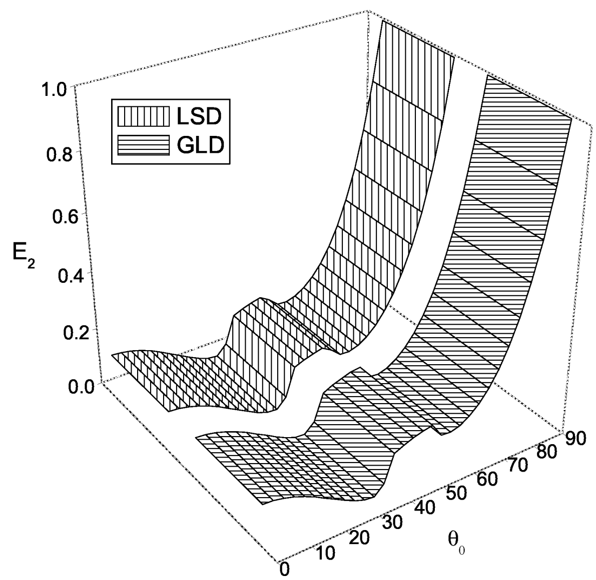

Fig. 10. Variations of energy ratio (E_2) with respect to angle of incidence (θ_0) for SV-wave.

Figure 11 indicates that the values of energy ratio E_{11} decrease continuously for both LSD and GLD cases. Figures 12 and 13 show that the values of energy ratios E_{22} and E_{33} oscillate up to a certain stage and after that

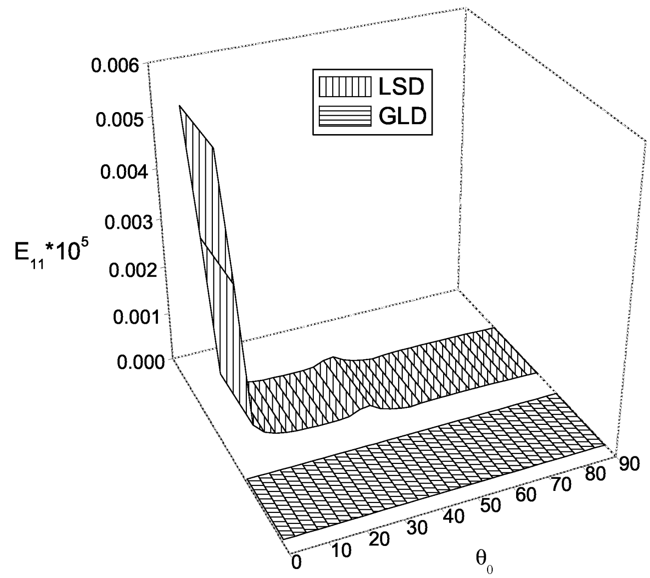

FIG. 11. Variations of energy ratio (E_{11}) with respect to angle of incidence (θ_0) for SV-wave.

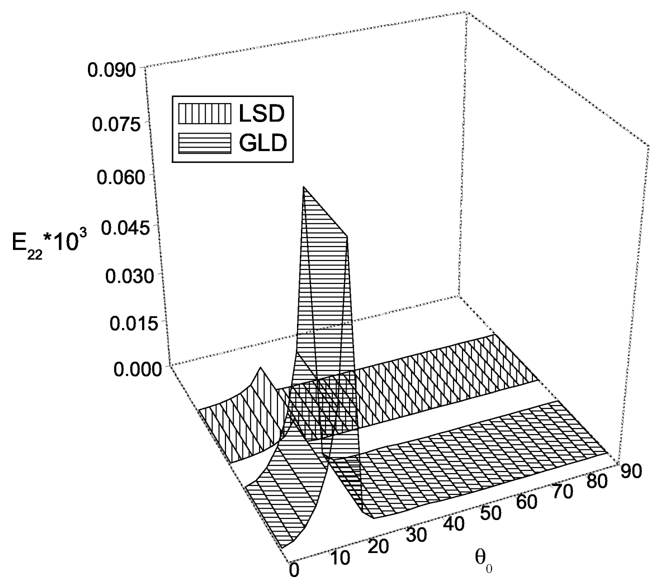

FIG. 12. Variations of energy ratio (E_{22}) with respect to angle of incidence (θ_0) for SV-wave.

become very small. Due to small values of E_{11} and E_{22}, the values of E_{11} and E_{22} are magnified by 10^5 and 10^3, respectively. We notice from Figs. 14 and 15 that firstly, the values of energy ratios E_{44} and E_{RR} show fluctuating

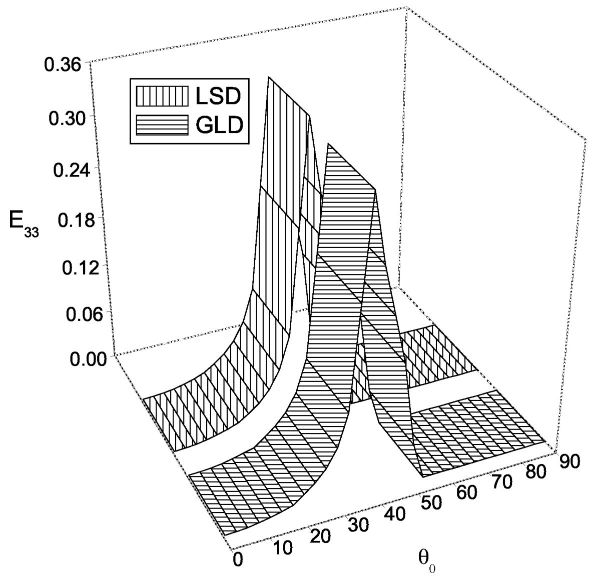

Fig. 13. Variations of energy ratio (E_{33}) with respect to angle of incidence (θ_0) for SV-wave.

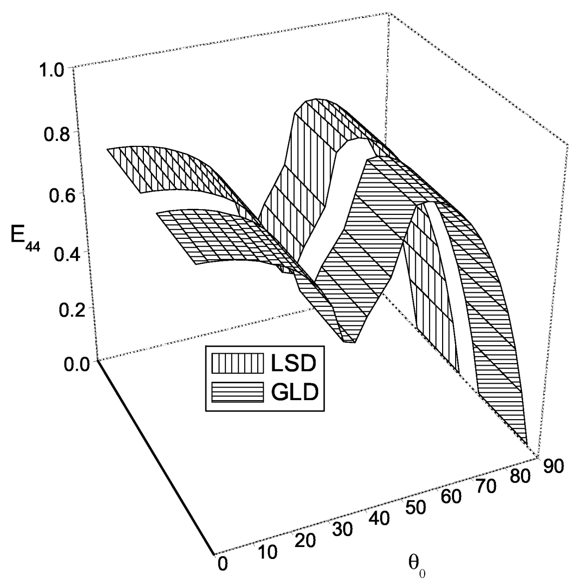

Fig. 14. Variations of energy ratio (E_{44}) with respect to angle of incidence (θ_0) for SV-wave.

behavior and then decrease continuously. Like in case of incident P-wave, the sum of all energy ratios is also found to be unity in case of incident SV-wave.

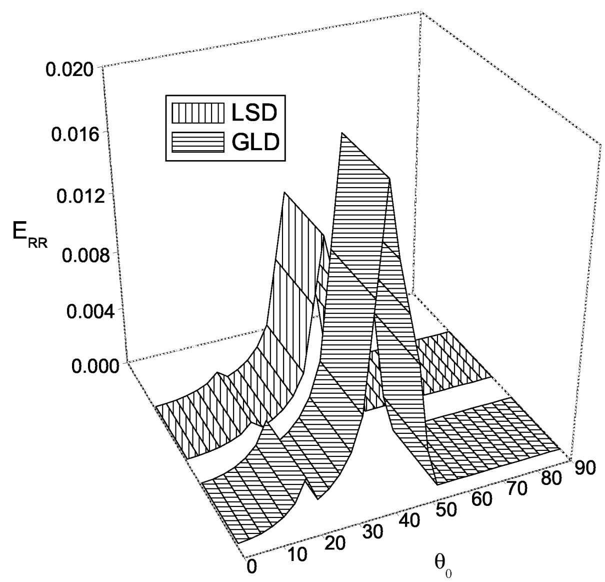

Fig. 15. Variations of energy ratio (E_{RR}) with respect to angle of incidence (θ_0) for SV-wave.

8. Conclusions

In the present article, the phenomenon of reflection and refraction of obliquely incident elastic waves at the interface between an elastic solid half-space and a thermoelastic diffusive solid half-space has been studied. The four waves in thermoelastic diffusive medium are identified and explained through different wave equations in terms of displacement potentials. Due to the presence of dissipation, the waves in thermoelastic diffusive medium are considered to be inhomogeneous waves. The energy ratios of different reflected and refracted waves to that of incident wave are computed numerically and presented graphically with respect to the angle of incidence.

From numerical results, we conclude that the effect of angle of incidence on the energy ratios of the reflected and refracted waves is significant. The sum of all energy ratios of the reflected waves, refracted waves and interference between refracted waves is verified to be always unity which ensures the law of conservation of incident energy at the interface.

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