General Steady-State Solution and Green's Functions in Orthotropic Piezothermoelastic Diffusion Medium

Rajneesh Kumar * and Vijay Chawla #

[Department of Mathematics, Kurukshetra University Kurukshetra-136119, Haryana (India)]

Abstract

The present investigation deals with the study of Green' functions in orthotropic piezothermoelastic diffusion material. With this objective, the two-dimensional general solution in orthotropic piezothermoelastic diffusion medium is derived at first. On the basis of the general solution, the Green' function for a point heat source and chemical potential source in the interior of semi-infinite orthotropic piezothermoelastic diffusion plane by introducing five newely harmonic functions. The components of displacement, stress, electric displacement, electric potential, temperature change and chemical potential are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, it is convenient to use. From the present investigation, a special case of interest is also deduced to depict the effect of diffusion. The components of stress, electric potential, temperature change and chemical potential are computed numerically and presented graphically.

Key Words: Green's function, piezothermoelasic diffusion, electric displacement, electric potential, semi- infinite.

1. Introduction

Green's functions or Fundamental solutions play an important role in both applied and theoretical studied on the physics of solids. Green's functions can be used to construct many analytical solutions solving boundary value problems of practical problems when boundary conditions are imposed. They are essential in boundary element method (BEM) as well as the study of cracks, defects and inclusion. Many researchers have been investigated the Green's function for elastic solid in isotropic and anisotropic elastic media, notable among them are Lord Kelvin [1], Freedholm [2], Synge [3], Pan and Chou[4], Deeg[5],wang[6] and Chen and Lin[7].

* Corresponding author, Tel: +91 9416120992

E-mail address: *rajneesh_kukmath@rediffmail.com,#vijay.chawla@ymail.com

Lee and Jiang [8] investigated the boundary integral formulation and two-dimensional fundamental solution for piezoelectric media. Wang and Zheng [9] derived the general solution for three-dimensional problem in piezoelectric media. Ding et al.[10] investigated the fundamental solution for piezoelectric media. Ding et al.[11] studied the fundamental solution for plane problem of piezoelectric materials.

The thermal effect is not considered in the above works. Rao and Sunar[12] pointed out the temperature variation in the piezoelectric media. Chen et al.[13] derived the general solution for transversely isotropic piezothermoelatic media. Chen et al.[14] obtained Green's function of transversely isotropic pyroelectric media with a penny shaped. Hou et al.[15] constructed Green's function for a point heat source on the surface of a semi-infinite transversely isotropic pyroelectric media.

Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region and it occurs in response to a concentration region and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal remains a practical process to separate isotopes of noble gases (e.g. xexon) and other light isotopes (e.g. carbon) for research purpose.

Nowacki [16-19] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. This implies infinite speed of propagation of thermoelastic waves . Sherief et al. [20] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Recently Kumar and Kansal [21] derived the basic equations for generalized thermoelastic diffusion (GL model) and discussed the Lamb waves. Kumar and Chawla[22] discussed the surface wave propagation in an elastic layer lying over a thermodiffusive elastic half-space with imperfect boundary. Kuang [23] discussed the variational principles for generalized thermodiffusion theory in pyroelectricity. Kumar and Chawla [24] obtained the fundamental solutions for orthotropic thermodiffusive elastic media. Recently Kumar and Chawla [25] derived the Green function for two-dimensional problem in orthotropic thermoelastic diffusion media. However, the important Green's function for two-dimensional problem for a steady point heat source in orthotropic piezothermoelastic diffusion medium has not been discussed so far.

The Green's function for two-dimensional in orthotropic piezothermoelastic diffusion medium is investigated in this paper. Based on the two-dimensional general solution of orthotropic thermoelastic diffusion media, the Green's function for a steady point heat source in the interior of semi-infinite orthotropic thermoelastic diffusion material is constructed by four newly introduced harmonic functions. A special case of interest is also deduced to depict the effect of diffusion.

2 Basic Equations

The basic governing equations of orthotropic piezothermodiffusive elastic materials can be found in Refs [23].If all the components are independent coordinate y, so called the plane problem. The constitutive equations in two-dimensional Cartesian coordinate (x, z) can be expressed as

$$\sigma_{xx} = c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \Phi}{\partial z} - \beta_1 T - b_1\mu \tag{1}$$

$$\sigma_{zz} = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \Phi}{\partial z} - \beta_3 T - b_3\mu \tag{2}$$

$$\sigma_{zx} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + e_{15}\frac{\partial \Phi}{\partial x} \tag{3}$$

$$D_x = e_{15}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \varepsilon_{11}\frac{\partial \Phi}{\partial x} \tag{4}$$

$$D_z = e_{31}\frac{\partial u}{\partial x} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \Phi}{\partial z} + p_3 T \tag{5}$$

where u and w are components of the mechanical displacement in x and z directions, respectively; σ_{ij} and D_i are the components of stress and electric displacement, respectively; β_i and b_i are material constants. Φ and T are electric potential and temperature increment, respectively; $c_{ij}, e_{ij}, \varepsilon_{ij}$, and p_3 are elastic piezoelectric, dielectric, thermal modules, diffusion modules and pyroelectric contants , respectively.

The mechanical, electric, heat equilibrium and mass diffusions equations for static problem, in the absence of body forces, free charges heat sources and mass diffusive sources

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{zx}}{\partial z} = 0, \qquad \frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0, \tag{6}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial \sigma_z}{\partial z} = 0, \tag{7}$$

$$\left(\lambda_1\frac{\partial^2}{\partial^2 x} + \lambda_3\frac{\partial^2}{\partial^2 z}\right)T = 0, \tag{8}$$

$$\left(D_1 \frac{\partial^2}{\partial^2 x} + D_3 \frac{\partial^2}{\partial^2 z}\right)\mu = 0 \tag{9}$$

We define the dimensionless quantities:

$$(x', z', u', w') = \frac{\omega_1^*}{v_1}(x, z, u, w), (\Phi', \sigma'_{ij}) = \frac{1}{\beta_1 T_0}\left(\frac{\omega^* e_{33}\Phi}{v_{10}}, \sigma_{ij},\right),$$

$$T' = \frac{T}{T_0},\ \mu' = \frac{\mu}{v_1^2}, D_i' = \frac{D_i}{\sqrt{\beta_1 T_0}}, H' = \frac{\beta_1 v_1}{c_{11} K_1 \omega_1^*} H$$

where

$$v_1^2 = \frac{\beta_1 T_0}{b_1},\ \omega_1^* = \frac{\beta_1 c_{11}}{b_1 K_1} \tag{10}$$

Substituting equation (1)-(5) into equations (6)-(7) and applying the dimensionless quantities defined by (10) on resulting equations, after suppressing the primes, we obtain

$$\left(\frac{\partial^2}{\partial x^2} + \delta_1 \frac{\partial^2}{\partial z^2}\right)u + \left(\delta_2 \frac{\partial^2}{\partial x \partial z}\right)w - e_1 \bar{\varepsilon}_p \frac{\partial^2 \Phi}{\partial x \partial z} - r_1\left(\frac{\partial}{\partial x}\right)T - q_1\left(\frac{\partial}{\partial x}\right)\mu = 0, \tag{11}$$

$$\left(\delta_2 \frac{\partial^2}{\partial x \partial z}\right)u + \left(\delta_1 \frac{\partial^2}{\partial x^2} + \delta_3 \frac{\partial^2}{\partial z^2}\right)w - \bar{\varepsilon}_p\left(e_2 \frac{\partial^2}{\partial x^2} + \delta_3 \frac{\partial^2}{\partial z^2}\right)\Phi - r_3\left(\frac{\partial}{\partial z}\right)T - q_3\left(\frac{\partial}{\partial z}\right)\mu = 0, \tag{12}$$

$$\left(e_1 \frac{\partial^2}{\partial x \partial z}\right)u + \left(e_2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)w - \bar{\varepsilon}_q\left(\bar{\varepsilon} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\Phi + g_1\left(\frac{\partial}{\partial z}\right)T + h_1\left(\frac{\partial}{\partial z}\right)\mu = 0, \tag{13}$$

$$\left(\frac{\partial^2}{\partial^2 x} + \bar{\lambda} \frac{\partial^2}{\partial^2 z}\right)T = 0, \tag{14}$$

$$\left(\frac{\partial^2}{\partial^2 x} + \bar{D} \frac{\partial^2}{\partial^2 z}\right)\mu = 0 \tag{15}$$

$$(\delta_1, \delta_2, \delta_3) = \frac{1}{c_{11}}(c_{44}, c_{13} + c_{44}, c_{33}), (e_1, e_2) = \frac{1}{e_{33}}(e_{31} + e_{15}, e_{15}),$$

$$(r_1, r_3, q_1, q_3) = \frac{1}{c_{11}}\left(a_1 T_0, a_3 T_0, b_1 v_1^2, b_3 v_1^2\right), (\varepsilon, g_1) = \frac{1}{\varepsilon_{33}}\left(\varepsilon_{11}, P_3 T_0, b_3 v_1^2\right),$$

$$(\bar{\varepsilon}_p, \bar{\varepsilon}_q) = \frac{1}{v_1}\left(\frac{e_{33}\Phi_0\omega_1^*}{c_{11}}, \frac{\varepsilon_{33}\Phi_0\omega_1^*}{e_{33}}\right), \bar{\lambda} = \frac{\lambda_3}{\lambda_1}, \bar{D} = \frac{D_3}{D_1}, \Phi_0 = \frac{v_1\beta_1 T_0}{\omega^* e_{33}}$$

The equations (11)-(15) can be written as

$$D\{u, w, \Phi, T, \mu\}^t = 0 \tag{16}$$

where D is the differential operator matrix given by

$$\begin{bmatrix} \frac{\partial^2}{\partial x^2} + \delta_1 \frac{\partial^2}{\partial z^2} & \delta_2 \frac{\partial^2}{\partial x \partial z} & e_1 \bar{\varepsilon}_p \frac{\partial^2}{\partial x \partial z} & -r_1 \frac{\partial}{\partial x} & -q_1 \frac{\partial}{\partial x} \\ \delta_2 \frac{\partial^2}{\partial x \partial z} & \delta_1 \frac{\partial^2}{\partial x^2} + \delta_3 \frac{\partial^2}{\partial z^2} & \bar{\varepsilon}_p \left(e_2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} \right) \frac{\partial}{\partial z} & -r_3 \frac{\partial}{\partial z} & -q_3 \frac{\partial}{\partial z} \\ e_1 \frac{\partial^2}{\partial x \partial z} & e_2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2} & -\bar{\varepsilon}_q \left(\bar{\varepsilon} \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) & g_1 \frac{\partial}{\partial z} & h_1 \frac{\partial}{\partial z} \\ 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2} + \bar{a} \frac{\partial^2}{\partial x^2} \right) & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2} + \bar{D} \frac{\partial^2}{\partial x^2} \right) \end{bmatrix} \tag{17}$$

Equation (16) is a homogeneous set of differential equations in u, w, Φ, T, μ.The general solution by the operator theory as follows

$$u = A_{i1}F, \qquad w = A_{i2}F, \qquad \Phi = A_{i3}F, \qquad T = A_{i4}F \qquad \mu = A_{i5}F \, (i = 1,2,3,4,5) \tag{18}$$

The determinant of the matrix D is given as

$$|D| = \left(a\frac{\partial^6}{\partial z^6} + b\frac{\partial^6}{\partial x^2 \partial z^4} + c\frac{\partial^6}{\partial x^4 \partial z^2} + d\frac{\partial^6}{\partial x^6} \right) \times \left(\frac{\partial^2}{\partial x^2} + \bar{\lambda}\frac{\partial^2}{\partial z^2} \right)\left(\frac{\partial^2}{\partial x^2} + \bar{D}\frac{\partial^2}{\partial z^2} \right) \tag{19}$$

Where a, b, c, d are given in Appendix A. The function F in equation (18) satisfies the following homogeneous equation

$$|D|F = 0 \tag{20}$$

It can be seen that if i was set to 1 or 2 in equation (18),one can gets two sets of general solution with $P = 0, T = 0$ and $\mu = 0$, which are actually to those for pure elasticity (Elliott [26]; and Ding et al [27]); $i = 1,2$ and 3 correspondence to the solution for piezoelectric discussed by (Ding and Liang [28]); $i = 4$ correspondence to the general solution W_1 (say) with $\mu = 0$ which is identical to that for piezothermoelaticity. Taking $i = 5$ correspondence to the general solution W_2 (say) with $T = 0$.

Due to the linear nature of the piezothermoelastic diffusion theory adopting in this paper, follows the same procedure as adopted by Xiang et el.[29,30] superposing W_1 and W_2 leads to

$$u = \left(a_1 \frac{\partial^6}{\partial x^6} + b_1 \frac{\partial^6}{\partial x^4 \partial z^2} + c_1 \frac{\partial^6}{\partial x^2 \partial z^4} + d_1 \frac{\partial^6}{\partial z^6} \right) \frac{\partial F}{\partial x}, \tag{21a}$$

$$w = \left(a_2 \frac{\partial^6}{\partial x^6} + b_2 \frac{\partial^6}{\partial x^4 \partial z^2} + c_2 \frac{\partial^6}{\partial x^2 \partial z^4} + d_2 \frac{\partial^6}{\partial z^6} \right) \frac{\partial F}{\partial z}, \tag{21b}$$

$$\Phi = \left(a_3 \frac{\partial^6}{\partial x^6} + b_3 \frac{\partial^6}{\partial x^4 \partial z^2} + c_3 \frac{\partial^6}{\partial x^2 \partial z^4} + d_3 \frac{\partial^6}{\partial z^6} \right) \frac{\partial F}{\partial z}, \tag{21c}$$

$$T = \left(a_4 \frac{\partial^8}{\partial x^8} + b_4 \frac{\partial^8}{\partial x^6 \partial z^2} + c_4 \frac{\partial^8}{\partial x^4 \partial z^4} + d_4 \frac{\partial^8}{\partial x^6 \partial z^2} + l_5 \frac{\partial^8}{\partial z^8} \right) F, \tag{21d}$$

$$\mu = \left(a_5 \frac{\partial^8}{\partial x^8} + b_5 \frac{\partial^8}{\partial x^6 \partial z^2} + c_5 \frac{\partial^8}{\partial x^4 \partial z^4} + d_5 \frac{\partial^8}{\partial x^6 \partial z^2} + l_6 \frac{\partial^8}{\partial z^8} \right) F, \tag{21e}$$

where the coefficients $a_k, b_k, c_k, d_k (k = 1,2,3,4,5)$ *and* l_5, l_6 are the expression given in appendix B.

The general solutions of equations of (16) in terms of F can be rewritten as

$$\prod_{j=1}^{5} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2} \right) F = 0 \tag{22}$$

Where

$z_j = s_j z,\ s_4 = \sqrt{\dfrac{\lambda_1}{\lambda_3}}, s_5 = \sqrt{\dfrac{D_1}{D_3}}$, and $s_j\,(j = 1,2,3)$ are three roots (with positive real part) of the following algebraic equation

$$as^6 - bs^4 + cs^2 - d = 0 \tag{23}$$

As known from the generalized **Almansi [31]** theorem, the function F can be expressed in terms of five harmonic functions

1 $F = F_1 + F_2 + F_3 + F_4 + F_5$ for distinct $s_j\,(j = 1,2,3,4,5)$

2 $F = F_1 + F_2 + F_3 + F_4 + zF_5$ for $s_1 \neq s_2 \neq s_3 \neq s_4 = s_5$

3 $F = F_1 + F_2 + F_3 + zF_4 + z^2 F_5$ for $s_1 \neq s_2 \neq s_3 = s_4 = s_5$

4 $F = F_1 + F_2 + zF_3 + z^2 F_4 + z^3 F_4$ for $s_1 \neq s_2 = s_3 = s_4 = s_5$

5 $F = F_1 + zF_2 + z^2 F_3 + z^3 F_4 + z^4 F_5$ for $s_1 = s_2 = s_3 = s_4 = s_5$ (24)

where F_j satisfies the following harmonic equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) F_j = 0 \qquad (j = 1,2,3,4,5) \tag{25}$$

The general solution for the case of distinct roots, can be derived as follows

$$u = \sum_{j=1}^{5} p_{1j} \frac{\partial^7 F_j}{\partial x \partial z_j^6},\ w = \sum_{j=1}^{5} s_j p_{2j} \frac{\partial^7 F_j}{\partial z_j^7},\quad \Phi = \sum_{j=1}^{5} s_j p_{3j} \frac{\partial^7 F_j}{\partial z_j^7}, T = p_{44} \frac{\partial^8 F_4}{\partial z_4^8}, \mu = p_{55} \frac{\partial^8 F_5}{\partial z_5^8}$$

(26)

Equation (26) can be further simplified by taking

$$p_{1j}\frac{\partial^6 F_j}{\partial z_j^6}=\psi_j \tag{27}$$

Making use of (27) in equation (26) yield

$$u=\sum_{j=1}^{5}\frac{\partial\psi_j}{\partial x},\quad w=\sum_{j=1}^{5}s_jP_{1j}\frac{\partial\psi_j}{\partial z_j},\ \Phi=\sum_{j=1}^{5}s_jP_{1j}\frac{\partial\psi_j}{\partial z_j},\ T=P_{34}\frac{\partial^2\psi_4}{\partial z_4^2},C=P_{45}\frac{\partial^2\psi_j}{\partial z_j^2} \tag{28}$$

where

$$P_{1j}=p_{2j}/p_{1j},\quad P_{2j}=p_{3j}/p_{1j},\quad P_{34}=p_{44}/p_{14},\quad P_{45}=p_{55}/p_{15},$$

The function ψ_j satisfies the harmonic equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\psi_j=0 \qquad j=1,2,3,4,5 \tag{29}$$

Applying the dimensionless quantities defined by (7) on equations (1)-(5), after suppressing the primes, with the aid of (28) we obtain

$$\sigma_{xx}=\sum_{j=1}^{5}\left(-f_1+f_2s_j^2P_{1j}+f_3s_j^2P_{2j}-P_{3j}-f_4P_{4j}\right)\frac{\partial^2\psi_j}{\partial z_j^2}, \qquad 30(a)$$

$$\sigma_{zz}=\sum_{j=1}^{5}\left(-f_2+f_5s_j^2P_{1j}+f_6s_j^2P_{2j}-f_7P_{3j}-f_8P_{4j}\right)\frac{\partial^2\psi_j}{\partial z_j^2}, \qquad 30\ (b)$$

$$\sigma_{zx}=\sum_{j=1}^{5}\left[f_9(1+P_{1j})+f_{10}P_{2j}\right]s_j\frac{\partial^2\psi_j}{\partial x\partial z_j} \qquad 30\ (c)$$

$$D_x = \sum_{j=1}^{5}\left[l_1(1+P_{1j}) - n_{10}P_{2j}\right]s_j \frac{\partial^2 \psi_j}{\partial x \partial z_j} \quad 30\text{ (d)}$$

$$D_z = \sum_{j=1}^{5}\left(-l_2 + l_3 s_j^2 P_{1j} - n_2 s_j^2 P_{2j} + n_3 P_{3j} - n_4 P_{4j}\right)\frac{\partial^2 \psi_j}{\partial z_j^2} \quad 30\text{ (e)}$$

where

$P_{31} = P_{32} = P_{33} = P_{35} = 0$ and $P_{41} = P_{42} = P_{43} = P_{44} = 0$

and

$$(f_1,, f_2, f_3, f_4, f_5, f_6,) = \frac{1}{a_1 T_0}\left(c_{11}, c_{13}, \frac{e_{31}\omega_1^* \Phi_0}{T_0}, b_1\mu_0, c_{33}, \frac{e_{33}\omega_1^* \Phi_0}{T_0}\right)$$

$$f_7 = \frac{a_3}{a_1},\ f_8 = \frac{b_3\mu_0}{b_1 T_0},\ f_9 = \frac{c_{44}}{a_1 T_0},\ f_{10} = \frac{e_{15}\omega_1^* \Phi_0}{a_1 T_0 v_1}$$

$$(l_1, l_2.l_3, n_1, n_2, n_3, n_4) = \frac{1}{\sqrt{a_1 T_0}}\left(e_{15}, e_{31}, e_{33}, \frac{\varepsilon_{11}\omega_1^* \Phi_0}{v_1}, \frac{\varepsilon_{11}\omega_1^* \Phi_0}{v_1}, P_3 T_0, b_3^* \mu_0\right),$$

Substituting equation (30) from into equation (1)-(5), with the aid of (5)-(6) gives

$$f_1 - (f_2 P_{1j} + f_3 P_{2j})s_j^2 + P_{3j} + f_4 P_{4j} = \left[f_9(1+P_{1j}) + f_{10}P_{2j}\right]s_j^2,$$

$$-f_2 + (f_5 P_{1j} + f_6 P_{2j})s_j^2 - f_7 P_{3j} - f_8 P_{4j} = \left[f_9(1+P_{1j}) + f_{10}P_{2j}\right],$$

$$-l_2 + (l_3 P_{1j} - n_2 P_{2j})s_j^2 + n_3 P_{3j} + n_4 P_{4j} = l_1(1+P_{1j}),$$

$$\left(a_1 - a_3 s_j^2\right)P_{3j} = 0$$

$$\left(D_1 - D_3 s_j^2\right) P_{4j} = 0 \qquad (j = 1,2,3,4,5) \tag{31}$$

By virtue of the above equations, the general solution (30) can be simplified as

$$\sigma_{xx} = -\sum_{j=1}^{5} s_j^2 w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \quad \sigma_{zz} = \sum_{j=1}^{5} w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \qquad \sigma_{zx} = \sum_{j=1}^{5} s_j w_{1j} \frac{\partial^2 \psi_j}{\partial x \partial z_j},$$

$$D_x = \sum_{j=1}^{5} s_j w_{2j} \frac{\partial^2 \psi_j}{\partial x \partial z_j}, D_z = \sum_{j=1}^{5} w_{2j} \frac{\partial^2 \psi_j}{\partial z_j^2} \tag{32}$$

where

$$w_{1j} = \frac{f_1 - (f_2 P_{1j} + f_3 P_{2j}) s_j^2 + P_{3j} + f_4 P_{4j}}{s_j^2} = f_9 (1 + P_{1j}) + f_{10} P_{2j} =$$

$$- f_2 + (f_5 P_{1j} + f_6 P_{2j}) s_j^2 - f_7 P_{3j} - f_8 P_{4j},$$

$$w_{2j} = -l_2 + (l_3 P_{1j} - n_2 P_{2j}) s_j^2 + n_3 P_{3j} + n_4 P_{4j} = l_1 (1 + P_{1j}) - n_1 P_{2j} \tag{33}$$

4. Green's Function for a point heat source in the interior of a semi-infinite orthotropic piezothermodiffusion elastic plane

As shown in Fig.1 We consider an orthotropic semi infinite piezothermodiffusion elastic plane $z \geq 0$. A point heat source H and chemical potential source P is applied at the point $(0, h)$ in two dimensional Cartesian coordinate (x, z) and the surface z=0 is free, thermally insulated and impermeable boundary. In Cartesian coordinate system, the general solution given by equation (28) and (32) in this semi-infinite plane is derived in this section.

In rest part of the paper, following notations are introduced

$$z_j = s_j z, \qquad h_k = s_k h, \qquad z_{jk} = z_j + h_k,$$

$$r_{jk} = \sqrt{x^2 + z_{jk}^2}, \qquad \bar{z}_{jk} = z_j - h_k, \qquad \bar{r}_{jk} = \sqrt{x^2 + \bar{z}_{jk}^2}, \qquad (j,k = 1,2,3,4) \tag{34}$$

By virtue of trial and error method, Green's function in the semi-infinite plane are assumed in the following form

$$\psi_j = A_j\left[\frac{1}{2}(\bar{z}_{jj}^2 - x^2)\left(\log\bar{r}_{jj} - \frac{3}{2}\right) - x\bar{z}_{jj}\tan^{-1}(\frac{x}{\bar{z}_{jj}})\right] + \sum_{k=1}^{5} A_{jk}\left[\frac{1}{2}(\bar{z}_{jk}^2 - x^2)\left(\log\bar{r}_{jk} - \frac{3}{2}\right) - xz_{jk}\tan^{-1}(\frac{x}{\bar{z}_{jk}})\right]$$

$$j = 1,2,3,4,5 \qquad (35)$$

where

A_j and $A_{jk}\,(j,k = 1,2,3,4,5)$ are thirty constants to be determined.

The boundary conditions at the surface $z = 0$ are

$$\sigma_{zz} = \sigma_{zx} = 0, \quad D_z = 0, \qquad \frac{\partial\mu}{\partial z} = 0, \qquad \frac{\partial T}{\partial z} = 0. \qquad (36)$$

Substituting Equation (30) in to equation (24) and (28), we obtain

$$u = \sum_{j=1}^{5} A_j\left[x(\log\bar{r}_{jj} - 1) + \bar{z}_{jj}\tan^{-1}\frac{x}{\bar{z}_{jj}}\right] - \sum_{j=1}^{5}\sum_{k=1}^{5} A_{jk}\left[x(\log r_{jk} - 1) + z_{jk}\tan^{-1}\frac{x}{z_{jk}}\right], \qquad (37a)$$

$$w = \sum_{j=1}^{5} s_j P_{1j} A_j\left[\bar{z}_{jj}(\log\bar{r}_{jj} - 1) - x\tan^{-1}\frac{x}{\bar{z}_{jj}}\right] + \sum_{j=1}^{5}\sum_{k=1}^{5} s_j P_{1j} A_{jk}\left[z_{jk}(\log r_{jk} - 1) - x\tan^{-1}\frac{x}{z_{jk}}\right], \qquad (37b)$$

$$\Phi = \sum_{j=1}^{5} s_j P_{1j} A_j\left[\bar{z}_{jj}(\log\bar{r}_{jj} - 1) - x\tan^{-1}\frac{x}{\bar{z}_{jj}}\right] + \sum_{j=1}^{5}\sum_{k=1}^{5} s_j P_{1j} A_{jk}\left[z_{jk}(\log r_{jk} - 1) - x\tan^{-1}\frac{x}{z_{jk}}\right], \qquad (37c)$$

$$T = P_{34} A_4 \log\bar{r}_{44} + P_{34}\sum_{k=1}^{5} A_{4k} r_{4k}, \qquad (37d)$$

$$\mu = P_{35} A_5 \log\bar{r}_{55} + P_{45}\sum_{k=1}^{5} A_{5k} r_{5k}, \qquad (37e)$$

$$\sigma_{xx} = -\sum_{j=1}^{5} s_j^2 w_{1j} A_j \log \bar{r}_{jj} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j^2 w_{1j} A_{jk} \log r_{jk}, \tag{37f}$$

$$\sigma_{zz} = \sum_{j=1}^{5} w_{1j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} w_{1j} A_{jk} \log r_{jk}, \tag{37f}$$

$$\sigma_{zx} = -\sum_{j=1}^{5} s_j w_{1j} A_j \tan^{-1}\frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j w_{1j} A_{jk} \tan^{-1}\frac{x}{z_{jk}}, \tag{37g}$$

$$D_x = -\sum_{j=1}^{5} s_j w_{2j} A_j \tan^{-1}\frac{x}{\bar{z}_{jj}} - \sum_{j=1}^{5}\sum_{k=1}^{5} s_j w_{2j} A_{jk} \tan^{-1}\frac{x}{z_{jk}}, \tag{37i}$$

$$D_z = \sum_{j=1}^{5} w_{2j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} w_{2j} A_{jk} \log r_{jk}, \tag{37j}$$

Considering the continuity on plane $z = h$ for w, Φ, τ_{zx} and D_x gives the following expressions

$$\sum_{j=1}^{5} s_j P_{1j} A_j = 0, \tag{38}$$

$$\sum_{j=1}^{5} s_j w_{1j} A_j = 0, \tag{39}$$

$$\sum_{j=1}^{5} s_j P_{2j} A_j = 0, \tag{40}$$

$$\sum_{j=1}^{5} s_j w_{2j} A_j = 0, \tag{41}$$

Equations (38)-(41) can be written in combined form as

$$\sum_{j=1}^{5} s_j P_{mj} A_j = 0, \qquad \text{(m=1, 2)} \tag{42}$$

$$\sum_{j=1}^{5} s_j w_{mj} A_j = 0, \tag{43}$$

Substitution $w_{mj}\,(m=1,2)$ from equation (33) in to (43) gives

$$\sum_{j=1}^{5} s_j \left[f_9 (1+P_{1j}) + f_{10} P_{2j} \right] A_j = 0, \tag{44}$$

$$\sum_{j=1}^{5} s_j \left[l_1 (1+P_{1j}) - n_1 P_{2j} \right] A_j = 0, \tag{45}$$

By virtue of the equations (42),(44)and (45) can be simplified to one equation

$$\sum_{j=1}^{5} s_j A_j = 0 \tag{46}$$

Considering the mechanical, electric, thermal equilibrium and chemical potential per unit mass for a rectangle of $0 \le z \le a$ and $-b \le x \le b\,(b>0)$, four equations can be obtained

$$\int_{-b}^{b} \sigma_{zz}(x,a)dx + \int_{0}^{a} \left[\sigma_{zx}(b,z) - \sigma_{zx}(-b,z) \right] dz = 0, \tag{47a}$$

$$\int_{-b}^{b} D_z(x,a)dx + \int_{0}^{a} \left[D_x(b,z) - D_x(-b,z) \right] dz = 0, \tag{47b}$$

$$-\bar{\lambda} \int_{-b}^{b} \frac{\partial T}{\partial z}(x,a)dx - \int_{0}^{a} \left[\frac{\partial T}{\partial x}(b,z) - \frac{\partial T}{\partial x}(-b,z) \right] dz = H \tag{47c}$$

$$-\bar{D} \int_{-b}^{b} \frac{\partial \mu}{\partial z}(x,a)dx - \int_{0}^{a} \left[\frac{\partial \mu}{\partial x}(b,z) - \frac{\partial \mu}{\partial x}(-b,z) \right] dz = P \tag{47d}$$

Some useful integrals are listed as follows

$$\int \log \bar{r}_{jj} = x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1}\left(\frac{x}{\bar{z}_{jj}}\right), \tag{48a}$$

$$\int \log r_{jk} = x(\log r_{jk} - 1) + z_{jk} \tan^{-1}(\frac{x}{z_{jk}}),$$

(48b)

$$\int \tan^{-1}(\frac{x}{\bar{z}_{jj}}) = \frac{1}{s_j}\left(x \log \bar{r}_{jj} + \bar{z}_{jj} \tan^{-1}(\frac{x}{\bar{z}_{jj}}) \right),$$

(48c)

$$\int \tan^{-1}(\frac{x}{z_{jk}}) = \frac{1}{s_j}\left(x \log r_{jk} + z_{jk} \tan^{-1}(\frac{x}{\bar{z}_{jj}}) \right),$$

(48d)

$$\int \frac{\partial T}{\partial z} dx = s_4 k_{34}\left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_{4k} \tan^{-1} \frac{x}{z_{4k}} \right),$$

(48e)

$$\int \frac{\partial T}{\partial x} dz = -\frac{k_{34}}{s_4}\left(A_4 \tan^{-1} \frac{x}{\bar{z}_{44}} + \sum_{k=1}^{4} A_4 \tan^{-1} \frac{x}{z_4} \right),$$

(48f)

$$\int \frac{\partial \mu}{\partial z} dx = A_j s_j p_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} + \sum_{k=1}^{4} A_{jk} s_j p_{2j} \tan^{-1} \frac{x}{z_{jk}},$$

(48g)

$$\int \frac{\partial \mu}{\partial x} dz = \frac{A_j}{s_j} p_{2j} \tan^{-1} \frac{x}{\bar{z}_{jj}} - \sum_{k=1}^{4} \frac{A_{jk}}{s_j} p_{2j} \tan^{-1} \frac{x}{z_{jk}}$$

(48h)

It is noticed that the integrals (48 f, h) is not continuous at $z = h$, following expression should be used

$$\int_{a_1}^{a_2} \frac{\partial T}{\partial x} dz = \int_{a_1}^{h^-} \frac{\partial T}{\partial x} dz + \int_{h^+}^{a_2} \frac{\partial T}{\partial x} dz \qquad (49a)$$

$$\int_{a_1}^{a_2} \frac{\partial \mu}{\partial x} dz = \int_{a_1}^{h^-} \frac{\partial \mu}{\partial x} dz + \int_{h^+}^{a_2} \frac{\partial \mu}{\partial x} dz$$

(49b)

Substituting equation (37) into equation (47 a,b) and using the integrals (48 a,b), yields

$$\sum_{j=1}^{5} w_{mj} A_j I_1 + \sum_{j=1}^{5} w_{mj} \sum_{k=1}^{5} A_{jk} I_2 = 0 \quad (m = 1,2) \tag{50}$$

where

$$I_1 = \left[\left(x(\log \bar{r}_{jj} - 1) + \bar{z}_{jj} \tan^{-1}(\frac{x}{\bar{z}_{jj}}) \right)_{z=a_1}^{z=a_2} \right]_{x=-b}^{x=b} - \left[\left(x \log \bar{r}_{jj} + \bar{z}_{jj} \tan^{-1}(\frac{x}{\bar{z}_{jj}}) \right)_{x=-b}^{x=b} \right]_{z=a_1}^{z=a_2}$$

(51a)

$$I_2 = \left[\left(x(\log r_{jk} - 1) + z_{jk} \tan^{-1}(\frac{x}{z_{jk}}) \right)_{z=a_1}^{z=a_2} \right]_{x=-b}^{x=b} - \left[\left(x \log r_{jk} + z_{jk} \tan^{-1}(\frac{x}{z_{jk}}) \right)_{x=-b}^{x=b} \right]_{z=a_1}^{z=a_2}$$

(51b)

On simplify, we obtain $I_1 = 0$ and $I_2 = 0$

i.e. equations (50), and 47(a, b) are satisfied automatically.

Making use of equation (37d) in equation (47c), and using the integrals (48 e,f) with the aid of (49 a) and $s_4 = \sqrt{\lambda_1 / \lambda_3}$ in the resulting equation, we obtain

$$A_4 I_3 + \sum_{k=1}^{4} A_{4k} I_4 = \frac{H}{P_{34}\sqrt{\frac{\lambda_3}{\lambda_1}}}, \tag{53}$$

where

$$I_3 = -\left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} - \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=h^-} + \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{44}})\right)_{x=-b}^{x=b}\right]_{z=h^+}^{z=a_2}$$

(54a)

$$I_2 = \left[\left(\tan^{-1}(\frac{x}{z_{4k}})\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2} - \left[\left(\tan^{-1}(\frac{x}{z_{4k}})\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b}$$

(54b)

on solving the equations (54 a) and (54b), we obtain $I_3 = -2\pi$ and $I_4 = 0$

Thus A_4 can be determined from equation (53) and (54), as follows

$$A_4 = -\frac{H}{2\pi P_{34}\sqrt{\frac{\lambda_3}{\lambda_1}}}$$

(55)

Substituting the value of μ from equation (37e) in equation (47d), and using the integrals (48 g,h) with the aid of (49 d) and $s_5 = \sqrt{D_1 / D_3}$ in the resulting equation, we obtain

$$A_5 I_3 + \sum_{k=1}^{5} A_{5k} I_4 = \frac{P}{P_{45}\sqrt{\frac{D_3}{D_1}}},$$

(56)

where

$$I_5 = -\left[\left(\tan^{-1}(\frac{x}{\bar{z}_{55}})\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} - \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{55}})\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=h^-} + \left[\left(\tan^{-1}(\frac{x}{\bar{z}_{55}})\right)_{x=-b}^{x=b}\right]_{z=h^+}^{z=a_2} = -2\pi$$

(57a)

$$I_2 = \left[\left(\tan^{-1}\left(\frac{x}{z_{5k}}\right) \right)_{x=-b}^{x=b} \right]_{z=a_1}^{z=a_2} - \left[\left(\tan^{-1}\left(\frac{x}{z_{5k}}\right) \right)_{z=a_1}^{z=a_2} \right]_{x=-b}^{x=b} = 0$$

(57b)

Thus A_5 can be determined from equation (56) and (57), as follows

$$A_5 = -\frac{P}{2\pi P_{45}\sqrt{\frac{D_3}{D_1}}}$$

(58)

At the surface $z = 0$, equation (16) reduces to

$$z_j = 0, \qquad h_k = s_k h, \qquad z_{jk} = h_k,$$

$$r_{jk} = \sqrt{x^2 + h_k^2} \qquad , \bar{z}_{jk} = -h_k, \quad \bar{r}_{jk} = \sqrt{x^2 + h_k^2}\ .$$

(59)

Substituting equation (37) into boundary conditions (36) and with the aid of $s_4 = \sqrt{\lambda_1 / \lambda_3},$ $s_5 = \sqrt{D_1 / D_3}$ and equation (59), we obtain

$$-s_j w_j A_j + \sum_{k=1}^{5} s_k w_k A_{kj} = 0,$$

(60)

$$w_{mj} A_j + \sum_{k=1}^{5} w_k A_{kj} = 0, \qquad j = 1,2,3,4,5$$

(61)

$$A_4 - A_{44} = 0, \qquad A_{4k} = 0, \qquad m = 1,2$$

(62)

$$A_5 - A_{55} = 0, \qquad A_{5k} = 0, \qquad k = 1,2,3,4$$

(63)

Thus the thirty constant A_j and $A_{jk}\,(j,k=1,2,3,4,5)$ can be determined by thirty equations including equations (42), (46), (55), (58) and (60)-(63).

5. Special case: In the absence of chemical potential per unit mass, equations (37a)-(37j) reduce to

$$u=\sum_{j=1}^{4}A_j\left[x(\log\bar r_{jj}-1)+\bar z_{jj}\tan^{-1}\frac{x}{\bar z_{jj}}\right]-\sum_{j=1}^{4}\sum_{k=1}^{4}A_{jk}\left[x(\log r_{jk}-1)+z_{jk}\tan^{-1}\frac{x}{z_{jk}}\right], \tag{64a}$$

$$w=\sum_{j=1}^{4}s_jP_{1j}A_j\left[\bar z_{jj}(\log\bar r_{jj}-1)-x\tan^{-1}\frac{x}{\bar z_{jj}}\right]+\sum_{j=1}^{4}\sum_{k=1}^{4}s_jP_{1j}A_{jk}\left[z_{jk}(\log r_{jk}-1)-x\tan^{-1}\frac{x}{z_{jk}}\right], \tag{64b}$$

$$\Phi=\sum_{j=1}^{4}s_jP_{1j}A_j\left[\bar z_{jj}(\log\bar r_{jj}-1)-x\tan^{-1}\frac{x}{\bar z_{jj}}\right]+\sum_{j=1}^{4}\sum_{k=1}^{4}s_jP_{1j}A_{jk}\left[z_{jk}(\log r_{jk}-1)-x\tan^{-1}\frac{x}{z_{jk}}\right], \tag{64c}$$

$$T=P_{34}A_4\log\bar r_{44}+P_{34}\sum_{k=1}^{4}A_{4k}r_{4k}, \tag{64d}$$

$$\sigma_{xx}=-\sum_{j=1}^{4}s_j^2w_{1j}A_j\log\bar r_{jj}-\sum_{j=1}^{4}\sum_{k=1}^{4}s_j^2w_{1j}A_{jk}\log r_{jk}, \tag{64e}$$

$$\sigma_{zz}=\sum_{j=1}^{4}w_{1j}A_j\log\bar r_{jj}+\sum_{j=1}^{4}\sum_{k=1}^{4}w_{1j}A_{jk}\log r_{jk}, \tag{64f}$$

$$\sigma_{zx}=-\sum_{j=1}^{4}s_jw_{1j}A_j\tan^{-1}\frac{x}{\bar z_{jj}}-\sum_{j=1}^{4}\sum_{k=1}^{4}s_jw_{1j}A_{jk}\tan^{-1}\frac{x}{z_{jk}}, \tag{64g}$$

$$D_x=-\sum_{j=1}^{4}s_jw_{2j}A_j\tan^{-1}\frac{x}{\bar z_{jj}}-\sum_{j=1}^{4}\sum_{k=1}^{4}s_jw_{2j}A_{jk}\tan^{-1}\frac{x}{z_{jk}}, \tag{64h}$$

$$D_z = \sum_{j=1}^{4} w_{2j} A_j \log \bar{r}_{jj} + \sum_{j=1}^{4}\sum_{k=1}^{4} w_{2j} A_{jk} \log r_{jk}, \tag{64i}$$

$z_j = s_j z,\ s_4 = \sqrt{\frac{\lambda_1}{\lambda_3}}$, and $s_j\,(j=1,2,3)$ are three roots (with positive real part) of the equation (23).

Considering the continuity on plane $z = h$ for w, Φ, τ_{zx} and D_x and using equation (64)-(64f) with the aid of $s_4 = \sqrt{\frac{\lambda_1}{\lambda_3}}$ and $A_4 = -\frac{H}{2\pi P_{34}\sqrt{\frac{\lambda_3}{\lambda_1}}}$ gives the following expressions in the absence of diffusion

$$\sum_{j=1}^{4} s_j P_{mj} A_j = 0, \qquad (m=1, 2) \tag{65}$$

$$\sum_{j=1}^{4} s_j A_j = 0 \tag{66}$$

$$-s_j w_j A_j + \sum_{k=1}^{4} s_k w_k A_{kj} = 0, \tag{67}$$

$$w_{mj} A_j + \sum_{k=1}^{4} w_k A_{kj} = 0, \qquad j = 1,2,3,4 \tag{68}$$

$$A_4 - A_{44} = 0, \qquad A_{4k} = 0, \qquad m = 1,2 \qquad k = 1,2,3 \tag{69}$$

The twenty constants A_j and $A_{jk}\,(j,k = 1,2,3,4)$ can be determined by twenty equations including equations (65)- (69), (67) by using the method of crammer rule.

The above results are similar as obtained by Xiong et al [32].

6. Numerical Results and Discussion:

In order to determine the constants A_j, A_{jk} $(j,k = 1,2,3,4,5)$,the method of Crammer's rule has been used to solve the system of non-homogeneous equations. We have used the **MATLAB 7.04** software TO computing the values of A_j, A_{jk} $(j,k = 1,2,3,4,5)$ for computer programme.

The material chosen for numerical calculations is Cadmium Selenide (Cdse), which is orthotropic material. The physical data for piezo-thermoelastic as given in Sharma [38]

$$c_{11} = 74.1\times10^9\,\mathrm{Nm^{-2}}, c_{12} = 45.2\times10^9\,\mathrm{Nm^{-2}}, c_{13} = 39.3\times10^9\,\mathrm{Nm^{-2}}\ c_{33} = 83.6\times10^9\,\mathrm{Nm^{-2}},$$

$$c_{44} = 13.2\times10^9\,\mathrm{Nm^{-2}}, T_0 = 298K,\ \ \beta_1 = 6.21\times10^5\,C^2/Nm^2, \beta_3 = 5.51\times10^5\,C^2/Nm^2,$$

$$K_1 = 9Wm^{-1}K^{-1}, K_3 = 7Wm^{-1}K^{-1}$$

$$e_{13} = -0.160\times10^{-3}cm^{-2},\ e_{33} = 34\times10^{-3}cm^{-2},\ \ e_{15} = -0.138\times10^{-3}cm^{-2}$$
$$\varepsilon_{11} = 8.26\times10^{-11}\,Nm^{-2}/K\ \ \varepsilon_{33} = 9.03\times10^{-11}\,Nm^{-2}/K,\ p_3 = -2.9\times10^{-6}\,cm^{-2}/K,$$

Behaviour of components of stress, electric potential, temperature change and chemical potential per unit mass

Figure (2)-(4) shows variation of components of displacement (T_{33}, T_{31}), electric potential (Φ) temperature change (T) and chemical potential per unit mass (μ) w.r.t. distance x. The without center symbol lines correspond to piezothermoelastic (PTE) and the centre symbol on these lines correspond to piezothermoelastic diffusion (PTDE).

Fig. 2 shows that the values of normal stress (T_{33}) increase for both cases PTE and PTDE. It is noticed that for smaller values of x, the values of T_{33} for the case of PTE remain more (in comparison PTDE), but for higher values of x reverse behavior occurs.

Fig. 3 shows that for smaller values of x, the values of tangential stress (T_{31}) for the cases of PTE (Z=5) and PTDE (Z=5) increase, but for higher values of x, it decreases whereas for the cases PTE (Z=10) and PTDE (Z=10), the values of T_{31} decrease initially, but for higher values of x, it increases. It is noticed that the values of T_{31} in case of PTDE remain more (in comparison with PTE).

Fig. 4 shows that for smaller values of x, the values of electric potential (Φ) slightly decrease for both cases PTE and PTDE, but for higher values of x, the values of Φ in case of PTE(z=5,z=10) decrease, whereas for the case of PTDE(z=5,z=10) ,it slightly increases. It is noticed that the values of Φ in case of PTE (z=10) remain more (in comparison with PTE (z=5), PTDE(z=5,z=10)).

Fig. 5 shows that the values of temperature change (T) increase for both cases PTE (z=5, z=10) for comparison, it is noticed that the values of T in case of PTE (z=10) remain more (in comparison with PTE z=5) for smaller values of x, but for higher values of x, the values of T in case of PTE (z=5) remain more, but there is minor difference in both values.

Fig. 6 shows that the values of chemical potential (μ) increase for both cases PTDE (Z=5,z=10) and for comparison it is noticed that the values of μ remain more in case of PTDE(z=10) (in comparison with z=5) for smaller values of x, but for higher values of x reverse behavior occur.

Conclusion: The Green's function for two dimensional problem in orthotropic piezothermoelastic diffusion madium has been derived. With this objective, the two-dimensional general solution in orthotropic piezothermodiffusion elastic medium has been derived at first. Based on the obtained two dimensional general solution, Green' functions for a point heat source and chemical potential source in the interior of semi-infinite orthotropic piezothermodiffusion elastic plane is constructed by five newely introduced harmonic functions. The components of displacement, stress, electric displacement, electric potential, temperature change and chemical potential are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, it is convenient to use. The components of displacement, electric potential, temperature change and chemical potential are computed numerically and depicted graphically. From the present investigation, a special case of interest is also deduced to depict the effect of diffusion. Significant diffusion effect is observed on components of stress and electric potential.

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Appendix A

$$a = -\delta_1(\varepsilon_p + \delta_3), \qquad b = \varepsilon_q(\delta_2^2 - \delta_3) - \delta_1(\delta_1\varepsilon_q + \delta_3\bar{\varepsilon}) - 2\delta_1\varepsilon_p e_2 + 2\delta_2\varepsilon_p e_1 - \varepsilon_p(1 + e_1\delta_3\varepsilon_p)$$

$$c = e_1\varepsilon_p(e_2\delta_2 - \delta_1 e_1) - \varepsilon_q(\delta_1 + \delta_3\bar{\varepsilon}) - 2\varepsilon_p e_2 - \delta_1\left(\delta_1\varepsilon_q\bar{\varepsilon} - \varepsilon_p e_2^2\right) + \delta_2(\delta_2\varepsilon_q\bar{\varepsilon} + e_1 e_2\varepsilon_p),$$

$$d = -(\varepsilon_p e_2^2 + \varepsilon_q\bar{\varepsilon}\delta_1)$$

Appendix B

$$a_1 = -q_1(\varepsilon_q\delta_1\bar{\varepsilon} + \varepsilon_p e_2^2), b_1 = \delta_2(\varepsilon_q q_3\bar{\varepsilon} - \varepsilon_p h_1 e_2) + \varepsilon_p e_1(\delta_1 h_1 + e_2 q_3) - \varepsilon_q q_1(\delta_1 + \delta_3\bar{\varepsilon}) - 2q_1 e_2\varepsilon_p -$$
$$\bar{a}q_1(\varepsilon_q\delta_1\bar{\varepsilon} + \varepsilon_p e_2^2) + \delta_2(r_3\varepsilon_q\bar{\varepsilon} - \varepsilon_p g_1 e_2) + \varepsilon_p e_1(g_1\delta_1 + e_2 r_3) - \varepsilon_q r_1(\delta_1 + \delta_3\bar{\varepsilon}) - 2r_1 r_2\varepsilon_p - \overline{D}\varepsilon_p r_1(\delta_1\bar{\varepsilon} + e_2^2)$$
$$c_1 = \delta_2(r_3\varepsilon_q - \varepsilon_p g_1) + e_1\varepsilon_p(g_1\delta_3 + r_3) - r_1(\varepsilon_q\delta_3 - \varepsilon_p) + \overline{D}[\delta_2\varepsilon_q(r_3\bar{\varepsilon} - g_1 e_2) + \varepsilon_p e_1(\delta_1 g_1 + e_2 r_3) - r_1(\varepsilon_q\delta_3 - \varepsilon_p)$$
$$+ \delta_2(q_3\varepsilon_q - h_1\varepsilon_p) + 2e_1\varepsilon_p h_1\delta_3 - q_1(\varepsilon_q\delta_3 + \varepsilon_p) + e_1 q_3\varepsilon_p] + a[\delta_2(q_3\varepsilon_p\bar{\varepsilon} - \varepsilon_p h_1 e_2) + \varepsilon_p e_1(\delta_1 h_1 + q_3 e_2) - 2q_1\varepsilon_p e_2 -$$
$$\varepsilon_q(q_1\delta_1 + \delta_3 q\bar{\varepsilon})]$$
$$d_1 = \overline{D}[\delta_2(\varepsilon_q r_3 - \varepsilon_p g_1) + \varepsilon_p e_1(g_1\delta_3 + r_3) - r_1(\varepsilon_q\delta_3 + \varepsilon_p)] + \bar{a}[\delta_2(\varepsilon_q q_3\bar{\varepsilon} - \varepsilon_p h_1 e_2) + \varepsilon_p e_1(\delta_1 h_1 + e_2 q_3) -$$
$$\varepsilon_q q_1(\delta_1 + \delta_3\bar{\varepsilon}) - 2q_1\varepsilon_p e_2$$

$$a_2 = \varepsilon_p e_2(g_1 + e_1 r_1) + \varepsilon_q\bar{\varepsilon}(r_1\delta_2 - r_3) + \varepsilon_p(h_1 e_2 - q_3\bar{\varepsilon}) + q_1(\varepsilon_q\delta_2\bar{\varepsilon} + e_1 e_2\varepsilon_p)$$
$$b_2 = \overline{D}[\varepsilon_p e_2(g_1 + e_1 r_1) + \varepsilon_q\bar{\varepsilon}(r_1\delta_2 - r_3)] + \varepsilon_p g_1(1 + \delta_1 e_2) - \varepsilon_q r_3(1 + \delta_1\bar{\varepsilon}) - e_1(g_1\delta_2\varepsilon_p - e_1 r_3) + r_1(\varepsilon_q\delta_2 + e_1\varepsilon_p) +$$
$$\bar{a}[\varepsilon_p(h_1 e_2 - q_3\bar{\varepsilon}) + q_1(\varepsilon_q\delta_2 + e_1 e_2\varepsilon_p)] + \varepsilon_p h_1(1 + \delta_1 e_2) - q_3(\varepsilon_p - \varepsilon_q\delta_1\bar{\varepsilon}) - e_1\varepsilon_p(h_1\delta_2 - e_1 q_3) + q_1(\varepsilon_q\delta_2 + e_1\varepsilon_p)$$
$$c_2 = \overline{D}[\varepsilon_p g_1(1 + \delta_1 e_2) - \varepsilon_q r_3(1 + \delta_1\bar{\varepsilon})] + \delta_2(\varepsilon_q r_1 - g_1 e_1\varepsilon_p) + e_1(\varepsilon_p r_1 - e_1 r_3)] + \delta_1(\varepsilon_p g_1 - \varepsilon_q r_3) + \bar{a}[\varepsilon_p h_1(1 + \delta_1 e_2) - q_3$$

$$(\varepsilon_p + \varepsilon_q \delta_1 \bar{\varepsilon}) - e_1 \varepsilon_p (\delta_2 h_1 + e_1 q_3) + q_1 (\varepsilon_p e_1^2 + \varepsilon_q \delta_2)] + \varepsilon_p \delta_1 (h_1 - q_3)$$

$$d_2 = \overline{D} \delta_1 (\varepsilon_p g_1 - \varepsilon_q r_3) + \bar{a} \delta_1 (\varepsilon_p h_1 - \varepsilon_q q_3)$$

$$a_3 = e_2 (r_1 \delta_2 - r_3) - \delta_1 (g_1 + e_1 r_1) + e_2 (q_1 \delta_2 - q_3) - \delta_1 (h_1 + q_1 e_1),\ b_3 = \overline{D}[e_2 (r_1 \delta_2 - r_3) - \delta_1 (g_1 + e_1 r_1)]$$

$$\delta_2 (\delta_2 g_1 + r_3 e_1) + r_1 (\delta_2 - \delta_3 e_1) + g_1 (\delta_3 + \delta_1^2) + r_3 (1 + \delta_1 e_2) + a[e_2 (\delta_2 q_1 - q_3) - \delta_1 (h_1 + q_1 e_1)] + h_1 (\delta_2^2 - \delta_3) +$$

$$q_3 (\delta_2 e_1 - 1) - \delta_1 (\delta_1 h_1 + q_3 e_2) + q_1 (\delta_2 - \delta_3 e_1)$$

$$c_3 = \overline{D}[\delta_2 (\delta_2 g_1 + r_3 e_1) + r_1 (\delta_2 - \delta_3 e_1) - g_1 (\delta_3 + \delta_1^2) - r_3 (1 + \delta_1 e_2)] - \delta_1 (r_3 + \delta_3 g_1) + a[h_1 (\delta_2^2 - \delta_3) - q_3 (1 + \delta_1 e_2) -$$

$$\delta_1^2 h_1 + q_3 e_1 \delta_2 + q_1 (\delta_2 - \delta_3 e_1)] - \delta_1 (\delta_3 h_1 + q_3), \quad d_3 = -\overline{D} \delta_1 (r_3 + \delta_3 g_1) - \bar{a} \delta_1 (\delta_3 h_1 + q_3)$$

$$a_4 = d,\ b_4 = c + \bar{a} D,\ c_4 = b + c\overline{D},\ d_4 = a + b\overline{D},\ l_5 = a\overline{D}$$

$$a_5 = d,\ b_5 = c + \bar{a} d,\ c_5 = b + \bar{a} c,\ d_5 = a + \bar{a} b,\ l_6 = a\bar{a}$$

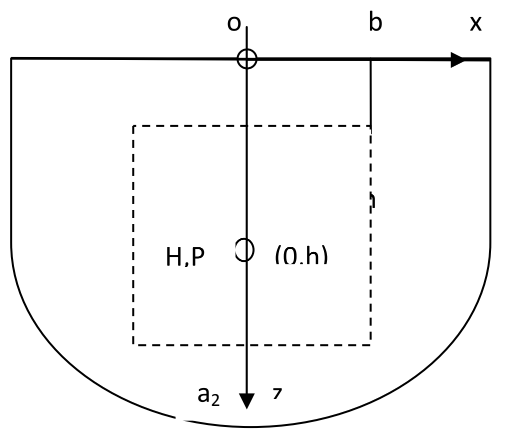

Fig. 1. A semi-infinite piezothermodiffusive elastic plane applied by a point heat source of strength H and chemical potential source of strength P

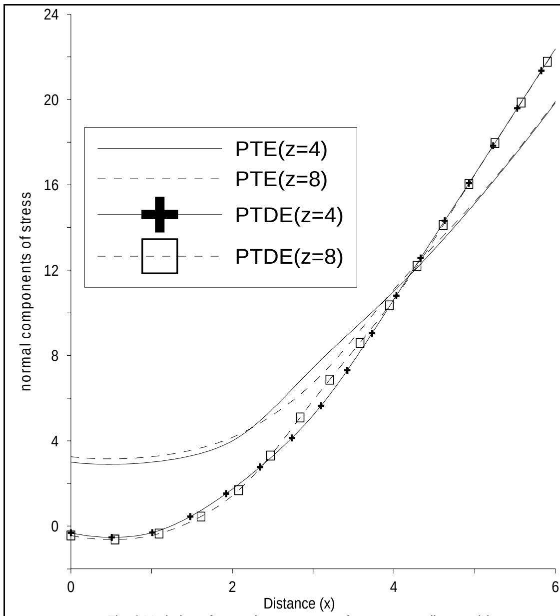

Fig. 2 Variation of normal components of stress w.r.t. distance(x)

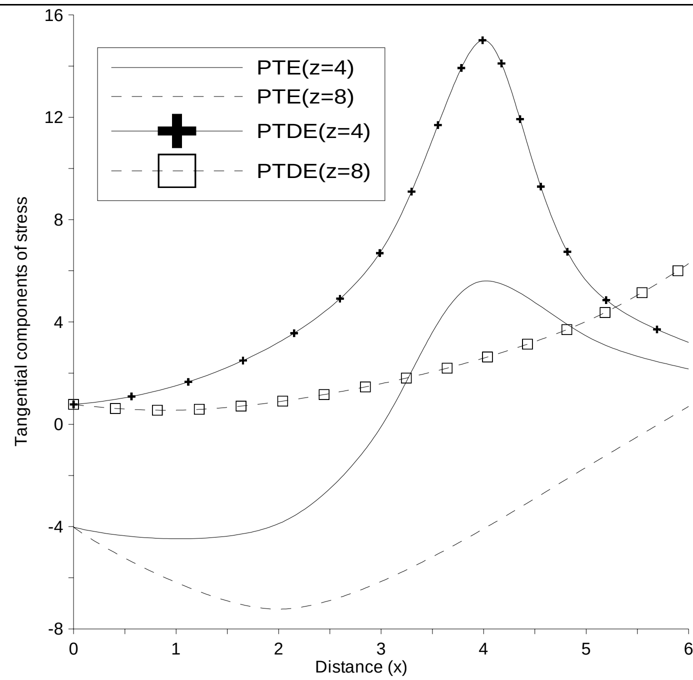

Fig.3 Variation of tangential components of stress (T_{31}) w.r.t. distance(x)

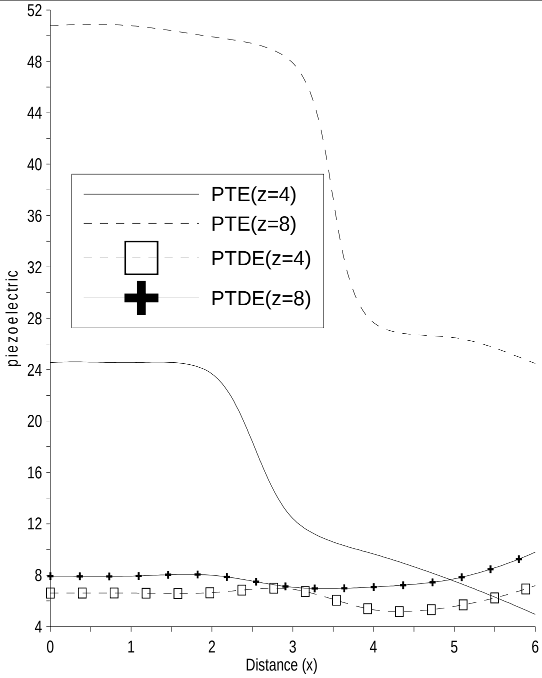

Fig.4 Variation of electric potential w.r.t. distance (x)

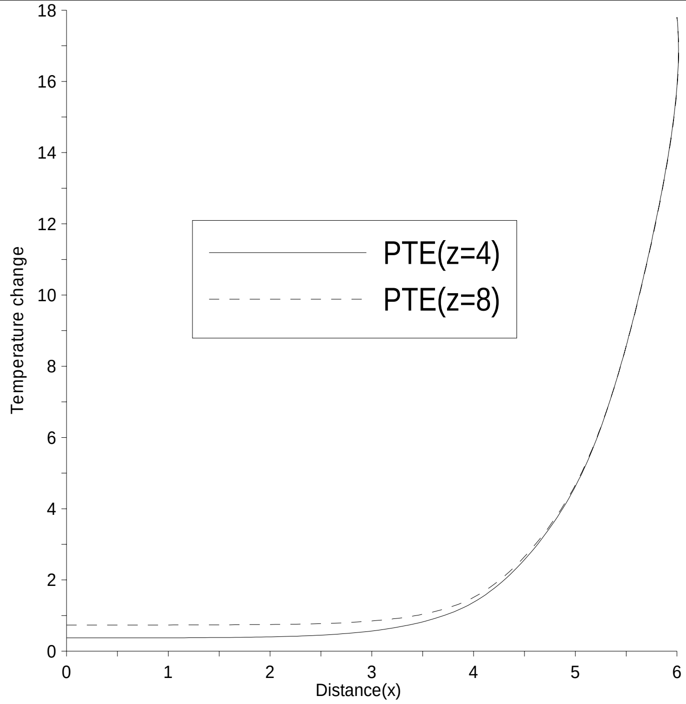

Fig. 5 Variation of Temperaure change (T) w.r.t. x

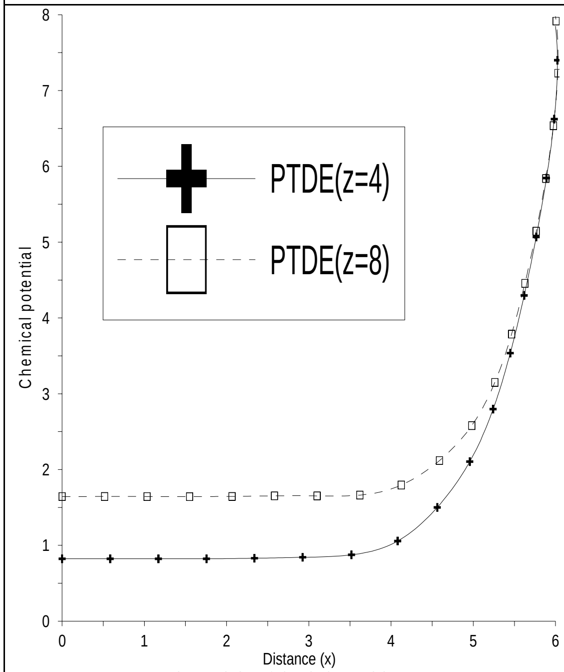

Fig.6 Variation of chemical potential w.r.t. x