

Answer for Reviewer B.

Dear Reviewer,

Thank you very much for your valuable remarks.

According to Your remarks:

“The authors treat an interesting problem. This problem was treated earlier by several authors (see for example papers in Journal Fractional calculus and Appl. Analysis). It may be helpful to look at the work: K.A.Lazopoulos,(2006), Nonlocal continuum mechanics and fractional calculus, Mechanics Research Communications, 33, 753-757. Author must comment on these works.”

I have decided to add additional references (cf. [24, 25, 26, 27, 28, 29, 30, 31]) giving at once comments on them, and making short comparison with my results (cf. Sec. 1 and Sec. 4.1.3.)

According to Your remark:

“The Section Remarks on objectivity is not satisfactory. The author must prove that the rigid body motion leads to zero strain tensor if body is subjected to rigid motion. Thus the tensor given by (4.15) must be zero, if the motion is given by (3.40).”

I have decided to give additional explanations in Sec. 4.1.2. below Eq. (4.15).

Answer for Reviewer C.

Dear Reviewer,

Thank you very much for your valuable remarks. I have taken them all under consideration.

According to Your remark 1:

“Page 2. The history of fractional calculus is well-known, Leibniz’s letter has been cited in the literature many times. Hence, this quotation should be omitted.”

I am sorry, but I have decided to leave this sentence. In my opinion it gives some ‘flavor’ of the paper. I hope that it is not a problem.

According to Your remark 2:

“Page 2. The author states: “There are infinitely many definitions of fractional derivatives...” There are several definitions, not infinitely many.”

I have changed this unclear sentence. Now we have: “There are many definitions of fractional derivatives...”.

According to Your remark 3:

“In Eq. (2.3) for the Riemann-Liouville fractional integral, the author uses the notation $I^\alpha f(t)$. There is no sense to use the notation $f^{(-1)}(t)$ for the n -fold integral instead of $I^n f(t)$.”

I have decided to change this notation according to Your suggestion.

According to Your remark 4:

“The well-known definition of the gamma function (Eq. (2.2)) should be deleted.”

I have deleted the definition of the gamma function.

According to Your remark 5:

“Formulae in Sections 3 and 4 look very nice but, unfortunately, they have one large disadvantage: they can be written only in Cartesian coordinates and cannot be written in curvilinear coordinates.

Let the author try to write down any of equations presented in Sections 3 and 4 (for example, equation (3.7) or (4.2)) in cylindrical coordinates.

Let the author try to write down the equilibrium equation in terms of displacement \mathbf{U} in cylindrical coordinates.

This is impossible.

To explain the problem consider the standard Laplace operator

$$\Delta = \text{div grad.}$$

This operator can be written in any curvilinear coordinate system, and in Cartesian coordinates

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \quad (*)$$

If we substitute the second derivatives in () by some fractional derivatives of the order*

$$1+\alpha \quad (0 < \alpha < 1)$$

$$\frac{\partial^{1+\alpha}}{\partial x^{1+\alpha}} + \frac{\partial^{1+\alpha}}{\partial y^{1+\alpha}} + \frac{\partial^{1+\alpha}}{\partial z^{1+\alpha}}, \quad (**)$$

*we do not obtain the fractional Laplace operator. The fractional Laplace operator was introduced by Riesz. It can be written in any curvilinear coordinate system, but its form is more complicated than (**).”*

I cannot agree with Your argumentation.

I agree of course that one should be able to rewrite the presented formulation in general coordinates. However not in a manner presented in Your argumentation. The situation is more complicated. I admit the possibility of non-standard definition of base vectors as well as allowing parameterization of length scale parameter. Even more, I think that we can do this using few concepts because it is not possible to reach consistent construction of a fractional vector calculus (cf. e.g. V.E. Tarasov Fractional Dynamics -> “Fractional Vector Calculus”).

Final results in general coordinates will result in a very complex relations (cf. Cottrill-Shepherd, K. and Naber, M., Fractional differential forms, Journal of Mathematical Physics, 45, 2, 2203-2212, 2001 or Chen, Y. and Yan, Z. and Zhang, H., Applications of fractional exterior differential in three-dimensional space, Applied Mathematics and Mechanics, 24, 3, 256-260, 2003). **and would make unclear the main context of this paper. So, we leave this problem as a future task.** (Notice that due to this reason none of cited papers dealing with similar concepts [24, 25, 26, 27, 28, 29, 30, 31] do not even mention about general coordinates and rarely explicitly show base vectors).

According to Your remark 6:

“It is clear from the above why the author considers a one-dimensional bar as an example. In my opinion, the strange oscillating behavior of displacements for small values of α (Figures 6 and 8) is due to large numerical errors of the numerical algorithm used in the paper and does not result from any mathematical reason.”

Up to now I have applied this formalism to the theory Kirchhoff-Love plates, plane problems of elasticity and to the 1D thermoelasticity and plasticity also.

All numerical results are improved using more general approach.

According to Your remark 7:

“English of the paper needs improvement”

I have once more revised all the text according the quality of English language.