Supplementary to the paper "Random composite: Stirred or shaken?"

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This Supplementary contains the effective conductivity $\sigma(x)$ in the form of polynomials in concentration x. The polynomials are obtained by use of the algorithm described in [5] and [21].

1 Polynomials for Random walks

In the present section, the polynomials $\sigma_t(x)$ are constructed by means of random walks described in the bulk of paper. The parameter $t = \frac{\tau}{30}$ corresponds to the number of random steps. The initial location $t = 0$ is the regular hexagonal array. Therefore, $\sigma_t(x)$ is the effective conductivity of the tth composite obtained after t steps. The constant 30 is chosen because the critical index s does not change for $t > 30$.

$\sigma_0(x) = 1 + 2x + 2x^2 + 2.01527x^3 + 1.94715x^4 + 2.00003x^5 + 2.08909x^6 + 2.18628x^7 + 2.27845x^8 + 2.38235x^9 + 2.51864x^{10} + 2.69506x^{11} + 2.90521x^{12} + 3.13231x^{13} + 3.35441x^{14} + 3.54969x^{15} + 3.70077x^{16} + 3.79717x^{17} + 3.83566x^{18}$

$\sigma_1(x) = 1 + 2x + 2x^2 + 4.22566x^3 + 0.44594x^4 + 1.76947x^5 + 2.81454x^6 + 2.88726x^7 + 2.3505x^8 + 1.75877x^9 + 1.37973x^{10} + 1.25739x^{11} + 1.38336x^{12} + 1.76542x^{13} + 2.41856x^{14} + 3.3303x^{15} + 4.43803x^{16} + 5.63428x^{17} + 6.79312x^{18}$

$\sigma_2(x) = 1+2x+2x^2+5.28295x^3+0.12249x^4+1.25038x^5+2.47388x^6+2.8672x^7+2.432x^8+1.7723x^9+1.3022x^{10}+1.11324x^{11}+1.19367x^{12}+1.55224x^{13}+2.22271x^{14}+3.21861x^{15} + 4.49348x^{16} + 5.93507x^{17} + 7.39212x^{18}$

$\sigma_3(x) = 1 + 2x + 2x^2 + 5.81369x^3 + 0.0483826x^4 + 0.955916x^5 + 2.15852x^6 + 2.77048x^7 + 2.49862x^8 + 1.86246x^9 + 1.35718x^{10} + 1.12896x^{11} + 1.17619x^{12} + 1.50538x^{13} + 2.15308x^{14} + 3.14323x^{15} + 4.44203x^{16} + 5.94468x^{17} + 7.49789x^{18}$

$\sigma_4(x) = 1 + 2x + 2x^2 + 6.2031x^3 + 0.000498947x^4 + 0.700659x^5 + 1.9423x^6 + 2.74014x^7 + 2.57728x^8 + 1.92835x^9 + 1.36567x^{10} + 1.08147x^{11} + 1.08129x^{12} +$

$1.37226x^{13} + 2.00498x^{14} + 3.03028x^{15} + 4.43917x^{16} + 6.13418x^{17} + 7.94643x^{18}$

$\sigma_5(x) = 1 + 2x + 2x^2 + 6.34488x^3 + 0.0695441x^4 + 0.824718x^5 + 1.8164x^6 + 2.46909x^7 + 2.34677x^8 + 1.8175x^9 + 1.35576x^{10} + 1.13586x^{11} + 1.17904x^{12} + 1.50374x^{13} + 2.15601x^{14} + 3.17046x^{15} + 4.52163x^{16} + 6.10585x^{17} + 7.76223x^{18}$

$\sigma_6(x) = 1 + 2x + 2x^2 + 6.54383x^3 + 0.013855x^4 + 0.68223x^5 + 1.70873x^6 + 2.49908x^7 + 2.4677x^8 + 1.91279x^9 + 1.37631x^{10} + 1.08731x^{11} + 1.07554x^{12} + 1.35622x^{13} + 1.98571x^{14} + 3.02412x^{15} + 4.47263x^{16} + 6.23952x^{17} + 8.15365x^{18}$

$\sigma_7(x) = 1 + 2x + 2x^2 + 6.63106x^3 + 0.0723371x^4 + 0.728637x^5 + 1.60721x^6 + 2.30895x^7 + 2.31593x^8 + 1.8625x^9 + 1.40899x^{10} + 1.16606x^{11} + 1.17809x^{12} + 1.47111x^{13} + 2.09855x^{14} + 3.1097x^{15} + 4.49661x^{16} + 6.16633x^{17} + 7.95562x^{18}$

$\sigma_8(x) = 1 + 2x + 2x^2 + 6.64703x^3 + 0.218954x^4 + 0.813384x^5 + 1.52769x^6 + 2.11113x^7 + 2.12465x^8 + 1.76112x^9 + 1.39623x^{10} + 1.21078x^{11} + 1.25717x^{12} + 1.57196x^{13} + 2.20881x^{14} + 3.20978x^{15} + 4.55954x^{16} + 6.16335x^{17} + 7.8634x^{18}$

$\sigma_9(x) = 1 + 2x + 2x^2 + 6.81958x^3 + 0.0936409x^4 + 0.700052x^5 + 1.47557x^6 + 2.16045x^7 + 2.23834x^8 + 1.85911x^9 + 1.43211x^{10} + 1.17729x^{11} + 1.16171x^{12} + 1.42511x^{13} + 2.03367x^{14} + 3.05643x^{15} + 4.50672x^{16} + 6.30275x^{17} + 8.27557x^{18}$

$\sigma_{10}(x) = 1 + 2x + 2x^2 + 6.64557x^3 + 0.543584x^4 + 0.863821x^5 + 1.35436x^6 + 1.82066x^7 + 1.87758x^8 + 1.64547x^9 + 1.39914x^{10} + 1.28788x^{11} + 1.37461x^{12} + 1.70835x^{13} + 2.3444x^{14} + 3.32054x^{15} + 4.61962x^{16} + 6.15156x^{17} + 7.76852x^{18}$

$\sigma_{11}(x) = 1+2x+2x^2+6.713x^3+0.60532x^4+0.788296x^5+1.22134x^6+1.70877x^7+1.8542x^8 + 1.70602x^9 + 1.50165x^{10} + 1.39682x^{11} + 1.4698x^{12} + 1.77715x^{13} + 2.37525x^{14} + 3.30368x^{15} + 4.55203x^{16} + 6.04043x^{17} + 7.63058x^{18}$

$\sigma_{12}(x) = 1 + 2x + 2x^2 + 6.77237x^3 + 0.609674x^4 + 0.768399x^5 + 1.17422x^6 + 1.65662x^7 + 1.82881x^8 + 1.7139x^9 + 1.52898x^{10} + 1.42784x^{11} + 1.49453x^{12} + 1.7905x^{13} + 2.37533x^{14} + 3.29227x^{15} + 4.53567x^{16} + 6.03001x^{17} + 7.63892x^{18}$

$\sigma_{13}(x) = 1 + 2x + 2x^2 + 6.66722x^3 + 1.00425x^4 + 0.763356x^5 + 1.01716x^6 +$

$1.42385x^7 + 1.63061x^8 + 1.62259x^9 + 1.54477x^{10} + 1.51485x^{11} + 1.61512x^{12} + 1.91605x^{13} + 2.48376x^{14} + 3.36618x^{15} + 4.56531x^{16} + 6.01696x^{17} + 7.59604x^{18}$

$\sigma_{14}(x) = 1 + 2x + 2x^2 + 6.68049x^3 + 1.06645x^4 + 0.729212x^5 + 0.963639x^6 + 1.37056x^7 + 1.59656x^8 + 1.61745x^9 + 1.56772x^{10} + 1.55983x^{11} + 1.67434x^{12} + 1.98058x^{13} + 2.54165x^{14} + 3.40089x^{15} + 4.55692x^{16} + 5.94644x^{17} + 7.4501x^{18}$

$\sigma_{15}(x) = 1 + 2x + 2x^2 + 6.60014x^3 + 1.27329x^4 + 0.75397x^5 + 0.937628x^6 + 1.30365x^7 + 1.50976x^8 + 1.54204x^9 + 1.52043x^{10} + 1.53947x^{11} + 1.67136x^{12} + 1.98593x^{13} + 2.55129x^{14} + 3.41796x^{15} + 4.59272x^{16} + 6.01888x^{17} + 7.57943x^{18}$

$\sigma_{16}(x) = 1 + 2x + 2x^2 + 6.43142x^3 + 1.91127x^4 + 0.577134x^5 + 0.634443x^6 + 0.990826x^7 + 1.33677x^8 + 1.56133x^9 + 1.68949x^{10} + 1.77631x^{11} + 1.89531x^{12} + 2.13832x^{13} + 2.6x^{14} + 3.36072x^{15} + 4.46443x^{16} + 5.89333x^{17} + 7.55657x^{18}$

$\sigma_{17}(x) = 1 + 2x + 2x^2 + 6.32191x^3 + 2.2594x^4 + 0.502057x^5 + 0.510662x^6 + 0.850653x^7 + 1.24453x^8 + 1.55491x^9 + 1.76098x^{10} + 1.88529x^{11} + 1.99225x^{12} + 2.18604x^{13} + 2.58273x^{14} + 3.28711x^{15} + 4.37255x^{16} + 5.85256x^{17} + 7.65834x^{18}$

$\sigma_{18}(x) = 1 + 2x + 2x^2 + 6.41815x^3 + 1.9904x^4 + 0.589701x^5 + 0.629316x^6 + 0.963049x^7 + 1.28681x^8 + 1.50649x^9 + 1.64984x^{10} + 1.7625x^{11} + 1.90637x^{12} + 2.16627x^{13} + 2.63483x^{14} + 3.39268x^{15} + 4.48523x^{16} + 5.89771x^{17} + 7.5437x^{18}$

$\sigma_{19}(x) = 1 + 2x + 2x^2 + 5.9658x^3 + 3.34963x^4 + 0.241689x^5 + 0.14861x^6 + 0.372336x^7 + 0.823755x^8 + 1.37986x^9 + 1.90747x^{10} + 2.30387x^{11} + 2.54541x^{12} + 2.71843x^{13} + 2.96924x^{14} + 3.43744x^{15} + 4.22385x^{16} + 5.3759x^{17} + 6.87209x^{18}$

$\sigma_{20}(x) = 1 + 2x + 2x^2 + 5.86584x^3 + 3.60184x^4 + 0.300137x^5 + 0.149423x^6 + 0.296597x^7 + 0.616542x^8 + 1.07144x^9 + 1.61966x^{10} + 2.19281x^{11} + 2.71481x^{12} + 3.15913x^{13} + 3.56291x^{14} + 3.9915x^{15} + 4.49799x^{16} + 5.10219x^{17} + 5.78762x^{18}$

$\sigma_{21}(x) = 1 + 2x + 2x^2 + 5.89479x^3 + 3.55363x^4 + 0.272497x^5 + 0.123334x^6 + 0.259417x^7 + 0.587222x^8 + 1.08423x^9 + 1.69439x^{10} + 2.32497x^{11} + 2.87996x^{12} + 3.32096x^{13} + 3.68033x^{14} + 4.02333x^{15} + 4.4059x^{16} + 4.85372x^{17} + 5.36008x^{18}$

$\sigma_{22}(x) = 1 + 2x + 2x^2 + 5.76914x^3 + 3.84644x^4 + 0.335175x^5 + 0.131605x^6 + 0.216452x^7 + 0.444658x^8 + 0.834302x^9 + 1.40486x^{10} + 2.12046x^{11} + 2.8742x^{12} + 3.54569x^{13} + 4.06509x^{14} + 4.42712x^{15} + 4.66499x^{16} + 4.81931x^{17} + 4.92034x^{18}$

$\sigma_{23}(x) = 1 + 2x + 2x^2 + 5.72548x^3 + 4.02672x^4 + 0.27242x^5 + 0.0773897x^6 + 0.140534x^7 + 0.350032x^8 + 0.752732x^9 + 1.37916x^{10} + 2.18434x^{11} + 3.02938x^{12} + 3.75311x^{13} + 4.2589x^{14} + 4.53921x^{15} + 4.64398x^{16} + 4.63783x^{17} + 4.57351x^{18}$

$\sigma_{24}(x) = 1 + 2x + 2x^2 + 5.74093x^3 + 4.02335x^4 + 0.263826x^5 + 0.0704172x^6 + 0.129078x^7 + 0.332607x^8 + 0.733131x^9 + 1.3651x^{10} + 2.18583x^{11} + 3.05408x^{12} + 3.79979x^{13} + 4.31503x^{14} + 4.58474x^{15} + 4.65802x^{16} + 4.60399x^{17} + 4.48234x^{18}$

$\sigma_{25}(x) = 1 + 2x + 2x^2 + 5.71926x^3 + 4.09008x^4 + 0.269143x^5 + 0.0740673x^6 + 0.13331x^7 + 0.332064x^8 + 0.717136x^9 + 1.32595x^{10} + 2.12446x^{11} + 2.98044x^{12} + 3.73027x^{13} + 4.26874x^{14} + 4.57961x^{15} + 4.70757x^{16} + 4.71571x^{17} + 4.65758x^{18}$

$\sigma_{26}(x) = 1 + 2x + 2x^2 + 5.6198x^3 + 4.29549x^4 + 0.257276x^5 + 0.0567061x^6 + 0.105105x^7 + 0.287864x^8 + 0.659685x^9 + 1.26741x^{10} + 2.08616x^{11} + 2.98028x^{12} + 3.76897x^{13} + 4.32958x^{14} + 4.63927x^{15} + 4.74705x^{16} + 4.72512x^{17} + 4.63559x^{18}$

$\sigma_{27}(x) = 1 + 2x + 2x^2 + 5.69279x^3 + 4.16187x^4 + 0.269813x^5 + 0.0684351x^6 + 0.120623x^7 + 0.306667x^8 + 0.677668x^9 + 1.28059x^{10} + 2.09215x^{11} + 2.98047x^{12} + 3.76768x^{13} + 4.32993x^{14} + 4.64x^{15} + 4.74272x^{16} + 4.70853x^{17} + 4.60034x^{18}$

$\sigma_{28}(x) = 1 + 2x + 2x^2 + 5.64233x^3 + 4.3598x^4 + 0.23572x^5 + 0.0391896x^6 + 0.0704153x^7 + 0.216985x^8 + 0.552916x^9 + 1.15464x^{10} + 2.02259x^{11} + 3.01699x^{12} + 3.91531x^{13} + 4.54082x^{14} + 4.83657x^{15} + 4.84953x^{16} + 4.67319x^{17} + 4.39914x^{18}$

$\sigma_{29}(x) = 1 + 2x + 2x^2 + 5.62476x^3 + 4.35116x^4 + 0.28237x^5 + 0.0627474x^6 + 0.0965688x^7 + 0.24123x^8 + 0.555628x^9 + 1.11775x^{10} + 1.94377x^{11} + 2.9167x^{12} + 3.8283x^{13} + 4.49799x^{14} + 4.85189x^{15} + 4.91879x^{16} + 4.7807x^{17} + 4.52581x^{18}$

$\sigma_{30}(x) = 1 + 2x + 2x^2 + 5.57857x^3 + 4.48064x^4 + 0.23922x^5 + 0.0374353x^6 + 0.0657429x^7 + 0.203934x^8 + 0.524662x^9 + 1.10981x^{10} + 1.97018x^{11} + 2.97236x^{12} + 3.89122x^{13} + 4.54243x^{14} + 4.86268x^{15} + 4.89615x^{16} + 4.73622x^{17} + 4.47527x^{18}$

$$\sigma_{60}(x) = 1 + 2x + 2x^2 + 5.37047x^3 + 5.23738x^4 + 0.139224x^5 + 0.0000127696x^6 + 0.000283076x^7 + 0.0282448x^8 + 0.176784x^9 + 0.59975x^{10} + 1.44073x^{11} + 2.6511x^{12} + 3.9372x^{13} + 4.9409x^{14} + 5.45189x^{15} + 5.46914x^{16} + 5.12824x^{17} + 4.59881x^{18}$$

$$\sigma_{90}(x) = 1 + 2x + 2x^2 + 5.27399x^3 + 5.54415x^4 + 0.116542x^5 + 0.00111247x^6 + 0.0015577x^7 + 0.00484347x^8 + 0.0770642x^9 + 0.370298x^{10} + 1.0932x^{11} + 2.29658x^{12} + 3.72437x^{13} + 4.96076x^{14} + 5.69907x^{15} + 5.8698x^{16} + 5.5901x^{17} + 5.04689x^{18}$$

$$\sigma_{120}(x) = 1 + 2x + 2x^2 + 5.14731x^3 + 5.83989x^4 + 0.103533x^5 + 0.00796939x^6 + 0.0149061x^7 + 0.00164948x^8 + 0.0207203x^9 + 0.217027x^{10} + 0.838171x^{11} + 2.01072x^{12} + 3.52074x^{13} + 4.92289x^{14} + 5.8418x^{15} + 6.15088x^{16} + 5.94484x^{17} + 5.41694x^{18}$$

$$\sigma_{150}(x) = 1 + 2x + 2x^2 + 5.23395x^3 + 5.71747x^4 + 0.130819x^5 + 0.00177325x^6 + 0.00524627x^7 + 0.0000101577x^8 + 0.0320389x^9 + 0.238918x^{10} + 0.8503x^{11} + 1.9827x^{12} + 3.44378x^{13} + 4.82722x^{14} + 5.78023x^{15} + 6.16896x^{16} + 6.0626x^{17} + 5.62847x^{18}$$

$$\sigma_{180}(x) = 1 + 2x + 2x^2 + 5.10286x^3 + 5.95404x^4 + 0.153951x^5 + 0.00127307x^6 + 0.00663742x^7 + 0.000797215x^8 + 0.0150509x^9 + 0.169971x^{10} + 0.699434x^{11} + 1.7627x^{12} + 3.22168x^{13} + 4.69119x^{14} + 5.7942x^{15} + 6.35026x^{16} + 6.388x^{17} + 6.05378x^{18}$$

$$\sigma_{210}(x) = 1 + 2x + 2x^2 + 5.11736x^3 + 5.94581x^4 + 0.119907x^5 + 0.00718393x^6 + 0.0186978x^7 + 0.0069819x^8 + 0.00578984x^9 + 0.146025x^{10} + 0.682789x^{11} + 1.78355x^{12} + 3.28818x^{13} + 4.77751x^{14} + 5.85708x^{15} + 6.35347x^{16} + 6.31565x^{17} + 5.90964x^{18}$$

$$\sigma_{240}(x) = 1 + 2x + 2x^2 + 5.05874x^3 + 6.09011x^4 + 0.13134x^5 + 0.00566237x^6 + 0.0181714x^7 + 0.00999888x^8 + 0.001225x^9 + 0.100984x^{10} + 0.558926x^{11} + 1.57891x^{12} + 3.0597x^{13} + 4.61599x^{14} + 5.84009x^{15} + 6.51478x^{16} + 6.64386x^{17} + 6.36494x^{18}$$

$$\sigma_{270}(x) = 1 + 2x + 2x^2 + 5.05874x^3 + 6.09011x^4 + 0.13134x^5 + 0.00566237x^6 + 0.0181714x^7 + 0.00999888x^8 + 0.001225x^9 + 0.100984x^(10) + 0.558926x^(11) + 1.57891x^(12) + 3.0597x^(13) + 4.61599x^(14) + 5.84009x^(15) + 6.51478x^(16) + 6.64386x^(17) + 6.36494x^18$$

$$\sigma_{300}(x) = 1 + 2x + 2x^2 + 5.13057x^3 + 5.9969x^4 + 0.124042x^5 + 0.00636917x^6 +$$

$0.0171998x^7+0.00799072x^8+0.00233233x^9+0.108784x^{10}+0.57375x^{11}+1.60082x^{12}+3.08862x^{13}+4.64661x^{14}+5.86027x^{15}+6.51069x^{16}+6.60646x^{17}+6.29273x^{18}$

2 Polynomial for Shaking model

The polynomial below is constructed by means of the shaken model introduced in [23], where each inclusion can randomly change its position within a small cell.

$\sigma_{shaken}(x) = 1+2x+2x^2+2.50496x^3+1.34794x^4+2.28669x^5+2.78469x^6+2.66817x^7+2.31341x^8+1.99453x^9+1.82074x^{10}+1.82006x^{11}+1.99754x^{12}+2.34983x^{13}+2.85936x^{14}+3.48676x^{15}+4.17211x^{16}+4.84582x^{17}+5.44337x^{18}$

3 Polynomial for the regular hexagonal array

The polynomial presented in this section is constructed by the exact formula for the effective conductivity [5]. The difference between the polynomials $\sigma_0(x)$ from Sec.1 and below demonstrates precision of the general approximate algorithm.

$\sigma_0(x) = 1+2x+2x^2+2x^3+2x^4+2x^5+2x^6+2.15084x^7+2.30169x^8+2.45253x^9+2.60338x^{10}+2.75422x^{11}+2.90507x^{12}+3.06744x^{13}+3.24119x^{14}+3.42632x^{15}+3.62283x^{16}+3.83071x^{17}+4.04997x^{18}+4.44142x^{19}+4.84599x^{20}+5.26454x^{21}+5.69792x^{22}+6.14699x^{23}+6.61261x^{24}+7.13504x^{25}+7.70007x^{26}$