

MHD natural convection in saturated porous media with  
heat generation/absorption and thermal radiation:

Closed-form solutions

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**Abstract**

The present work is devoted to the free convection flow occurring over a heated vertically stretching permeable thermally radiative magnetized surface placed in a porous medium under the influence of a temperature dependent internal heat generation or absorption. It is shown that the governing equations are reducible to a self-similar nonlinear ordinary differential equation of third order whose solutions are constructed analytically in the purely exponential series form. Under special circumstances, closed-form solutions are available which clearly indicate the existence of dual natural convection solutions.

Otherwise, analytical solutions are still possible which are shown to be computed from

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an elegant algorithm without a need to invoke any numerical means. Exact solutions demonstrate, in physical insight that, in the presence of a heat sink absorbing the temperature from the porous medium enhances the rate of heat transfer from the wall, whereas a heat source mechanism will surely overheat the system during the wall heating process, resulting in poor heat transfer rates. The presented exact solutions are beneficial for investigation of free convection phenomena in different geometries taking into account more complex physical features in higher dimensions.

**Key words:** Porous vertical wall, Magnetic Field, Radiative flux, Permeable surface, Heat generation/absorption, Rate of heat transfer, Analytical solutions.

## Nomenclature

### *Roman symbols*

$a$	positive temperature parameter
$(a_n, A_n)$	coefficients of the series
$B_0$	uniform magnetic field strength
$(c, d)$	radiative heat flux parameters
$c_p$	specific heat
$e$	constant
$f$	dimensionless self-similar velocity
$g$	gravitational acceleration
$k$	thermal conductivity
$K$	porosity parameter
$M$	magnetic interaction parameter

$Nu$	Nusselt number
$Nr$	radiation parameter
$s$	dimensionless suction or injection parameter
$q_r$	radiation flux term
$q'''$	temperature dependent heat flux term
$(\tilde{Q}, \tilde{\tilde{Q}})$	heat flux parameters
$Q$	heat generation/absorption parameter
$T$	temperature
$T_w$	surface temperature
$T_\infty$	far field temperature
$u$	velocity component in $x$ -direction
$v$	velocity component in $y$ - direction
$Uw$	wall stretching velocity
$v_w$	permeability parameter
$Vw$	dimensional wall mass transfer velocity
$(x, y)$	longitudinal and transverse directions

*Greek symbols*

$(\alpha, \beta)$	constants
$\alpha_m$	thermal diffusivity
$\beta_m$	thermal expansion parameter
$\eta$	scaled boundary layer coordinate
$\kappa$	fixed parameter
$\theta$	scaled temperature

$\mu$	dynamic viscosity
$\nu$	kinematic viscosity
$\sigma$	electrical conductivity
$\rho$	density
$\lambda$	positive exponent
$\Lambda$	temperature parameter

*Superscripts*

$-$	scaled quantities
$\sim$	scaled quantities

*Subscripts*

$l$	local quantities
$w$	quantities at wall
$\infty$	quantities at far field

*Acronyms*

MHD	Magnetohydrodynamics
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## 1. Introduction

It is now well-known that the density variations due to the temperature gradients in a fluid medium lead to a physical mechanism called free (natural) convection. Such a physical operation is encountered in many real-life applications as far as the porous media is concerned, for example while cooling the electronic equipments in computers, forming the clouds in atmosphere, interaction of spices in chemical engineering, processes in biological systems, removing heat from nuclear fuel debris, insulating the constructions, storing the energy and foods and

so on, refer to the comprehensive reviews [1], [2] and [3]. The current investigation is also about the phenomenon of MHD natural convection in saturated porous media with heat generation/absorption and thermal radiation with an objection of gaining closed-form solutions.

The heated vertical plate placed in a medium of porous feature received considerable attention lately in order to explain the process of free convection. In the absence of a source to cause a heat generation, several temperature conditions, from a variable surface temperature to a variable heat flux were successfully analyzed in [4]. Effects of porosity were numerically simulated in [5]. On the other hand, the internal heat generation has an active role in the natural convection heat transfer in many porous media problems. Due to its significance, [6] first considered the internal heat generation term in the governing energy equation as a fixed variable term for the problem of free convection from a vertical wall embedded in a porous medium. Many researchers adhered this plausible work, by taking into account such an internal heat generation term. For instance, [7] extended the work of [6] to the non-Newtonian fluid case. [8] incorporated the effects of mass injection/removal through the wall boundary. [9] discussed the convective boundary condition associated with the temperature. [10] took into account the double diffusive Soret and Dufour effects with a power-law fluid. The radiation effects were included in the numerical investigation of [11]. [12] studied the effects of viscous dissipation and magnetic field. The free convection problem affected by the presence of a saturated nanofluid medium was the focus of the study in [13]. The conditions of variable thermal diffusivity and mass diffusivity in a non-Newtonian fluid were examined in [14], followed by the concentration effects in [15].

The internal heat generation term introduced and employed in [6] to the heated vertical wall was also considered in the following applications, such as the natural convection phenomena

in vertical cones, see [16] and [17], and in the horizontal plates [18] and [19]. Very interesting applications of porous media correlated with natural convection can be found in the recent papers [20], [21], [22], [23] and [24].

The prime motivation for the current research is to substitute the fixed internal heat generation term as used in the above papers with that of a temperature dependent heat flux term representing more realistic situations. Within this perspective, the effects of such a term are to be investigated on the free convection taking place along a vertical plate embedded in a porous medium. The plate is assumed to be heated permeable and radiative subjected to a uniform magnetic field. The governing equations are reduced to a self-similar form whose solutions, unlike the existing numerical literature, are presented in an elegant analytic form which can be expressible in either closed-form or infinite series. The solutions perfectly conform with those numerical ones for the particular parameters. In the case of series, an algorithm is further introduced necessitating no numerical computations. The engineering interest of heat transfer rate can be easily derived from the presented data.

## 2. Physical problem and mathematical formulation

As depicted in figure 1, we consider the free convection phenomenon over a flat plate, heated with the wall temperature  $T_w(x) = T_\infty + ax^\Lambda$ , and embedded in a porous medium, taking into account a heating/cooling heat flux term dependent on the temperature as  $q''' = \tilde{Q}(T - T_w)$  in place of an exponentially decaying shape as introduced in [6]. The wall is assumed to be permeable with a wall transpiration velocity  $Vw(x) = v_w x^{\frac{\Lambda-1}{2}}$  and it is also radiative with a Rosseland based radiation flux  $q_r = -c(T^4)_y$ , but linearized such that  $q_r = dT_y$ . A uniform magnetic field further acts against the surface to retard the motion with a uniform magnetic

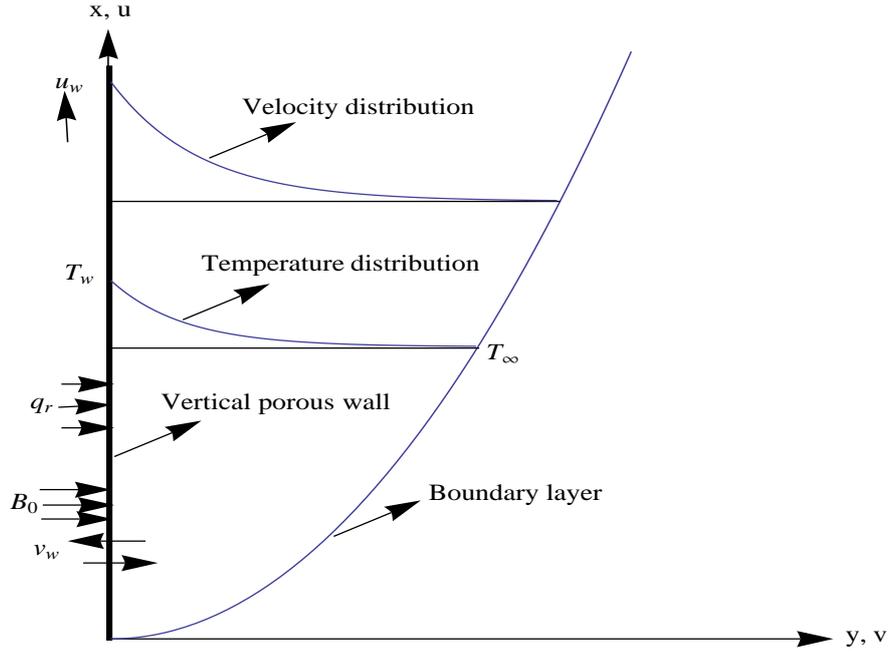


Figure 1: The schematic of the natural convection over a heated vertical flat surface subjected to various physical phenomena.

field strength  $B_0$ .

Following [6], the natural convection takes place according to the governing equations and boundary conditions

$$\begin{aligned}
 u_x + v_y &= 0, \\
 \left(1 + \frac{\sigma B_0^2 K}{\mu}\right) u &= \frac{gK\beta_m}{\nu}(T - T_\infty), \\
 uT_x + vT_y &= \frac{k}{\rho c_p} T_{yy} + \frac{\tilde{Q}}{\rho c_p}(T - T_\infty) + \frac{d}{\rho c_p} T_{yy}, \\
 v &= Vw(x), \quad T = Tw(x) \quad \text{on } y = 0, \\
 u &\rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty.
 \end{aligned} \tag{2.1}$$

With the help of the similarity transformations

$$\begin{aligned} u &= \alpha_m a e x^\Lambda f'(\eta), & v &= -\alpha_m \sqrt{\frac{ae}{4}} x^{\frac{\Lambda-1}{2}} [(1+\Lambda)f(\eta) + (\Lambda-1)\eta f'(\eta)], & T &= T_\infty + ax^\Lambda \theta \\ e &= \frac{gK\beta_m}{\alpha_m \nu}, & \alpha_m &= \frac{k}{\rho c_p}, & \eta &= \sqrt{ae} x^{\frac{\Lambda-1}{2}} y, \end{aligned} \quad (2.2)$$

system (2.1) is expressed in the reduced similarity form

$$\begin{aligned} f''' + \alpha f f'' - \beta f'^2 + Q f' &= 0, \\ f(0) = s, \quad f'(0) = 1, \quad f'(\infty) &= 0 \end{aligned} \quad (2.3)$$

with  $(1+M)f' = \theta$ . Here,  $v_w = -\alpha_m \sqrt{\frac{ae}{2}} (\Lambda+1) \frac{s}{1+M}$  and  $s > 0$  represents wall suction,  $s < 0$  wall injection. Moreover, the appearing parameters are

$$\alpha = \frac{1+\Lambda}{2(1+Nr)(1+M)}, \quad \beta = \frac{\Lambda}{(1+Nr)(1+M)},$$

with  $Nr = \frac{d}{k}$  being the thermal radiation parameter,  $Q = \frac{\tilde{Q}}{kae(1+Nr)}$  being the heat generation ( $> 0$ ) or absorption ( $< 0$ ) parameter ( $\tilde{Q} = \tilde{Q}x^{1-\Lambda}$ ) and  $M = \frac{\sigma B_0^2 K}{\mu}$  being the magnetic interaction parameter. We should note that replacing  $Qf'$  in (2.3) by the exponential form  $e^{-\eta}$  and also accounting for  $M = s = Nr = 0$ , (2.3) turns in to the mathematical model given in [6].

In terms of engineering viewpoint, we are mainly interested in the local rate of heat transfer, or the local Nusselt number defined by

$$Nu = \frac{Nu_l}{\sqrt{e a x^{\frac{\Lambda+1}{2}}}} = -\theta'(0) = -f''(0). \quad (2.4)$$

Integrating (2.3) once from the semi-infinite physical domain, we have

$$f'' + \alpha f f' + Q f + (\alpha + \beta) \int_\eta^\infty f'^2(\eta) d\eta = Q f_\infty, \quad (2.5)$$

where  $f_\infty = f(\infty)$  and so, from (2.4),

$$Nu = (\alpha + Q)s - Q f_\infty + (\alpha + \beta) \int_0^\infty f'^2(\eta) d\eta. \quad (2.6)$$

The result presented in (2.6) alone is successfully able to explain the effects of  $s$  and  $Q$ , for pre-assigned values of  $\alpha$  and  $\beta$  (preserving the effects of magnetic field, temperature and radiation) on the heat transfer analysis of the considered problem.

### 3. Analytical solution method

Dissimilar to the available numerical solutions in the literature obtained from various numerical schemes, we plan to find out exact solutions representing the flow and temperature fields. The following solution method stems from the study published in [25], particularly valid when  $Q < 0$  (in place of  $M$  in [25]). Taking into account the asymptotic far field condition of the system (2.3), it is realistic to search for solutions of the purely exponential serial form

$$f(\eta) = \sum_{n=0}^{\infty} A_n e^{-n\lambda\eta}, \quad (3.7)$$

such that the exponent  $\lambda$  in (3.7) is a positive constant to match to the infinity boundary condition and  $A_0 = \lim_{\eta \rightarrow \infty} f(\eta) = f(\infty) = f_{\infty}$ . Both  $A_0$  and  $\lambda$  are to be found.

Injecting (3.7) into (2.3) results in

$$-n\lambda(n^2\lambda^2 + Q)A_n + \alpha n^2\lambda^2 A_0 A_n + \sum_{k=0}^{n-1} k\lambda^2 [(\alpha - \beta)k + \beta(2k - n)] A_k A_{n-k} = 0, \quad n \geq 0 \quad (3.8)$$

There are two subcases to be considered now, in accordance with the far field behavior of the stream function  $f$ .

#### 3.1. $A_0 = f(\infty) = 0, A_1 \neq 0$

In this case the parameter  $Q$  can not be zero and we have

$$\lambda = \sqrt{-Q}$$

from (3.8) implying that (3.7) type solutions are available only when  $Q < 0$  corresponding to the presence of heat absorption, and hence cooling physical situation, to be determined from the system

$$\begin{aligned}
A_n &= \frac{1}{\lambda n(n^2 - 1)} \sum_{k=0}^{n-1} k[(\alpha - \beta)k + \beta(2k - n)]A_k A_{n-k}, \quad n \geq 2, \\
s &= \sum_{n=1}^{\infty} A_n, \\
1 &= \sum_{n=1}^{\infty} (-n\lambda)A_n.
\end{aligned} \tag{3.9}$$

Making use of (3.9), we can determine the pair of unknowns  $(\lambda, A_1)$ , numerically. On the other hand, to avoid the numerics, we propose the use of new coefficients  $a_n$  from

$$A_n = \lambda \kappa^n a_n, \tag{3.10}$$

with  $\kappa = \frac{A_1}{\lambda}$  such that  $a_1 = 1$ .

With the help of (3.10), (3.9) can be rewritten as

$$\begin{aligned}
a_0 &= 0, \\
a_1 &= 1, \\
a_n &= \frac{1}{n(n^2 - 1)} \sum_{k=0}^{n-1} k[(\alpha - \beta)k + \beta(2k - n)]a_k a_{n-k}, \quad n \geq 2, \\
\frac{s}{\lambda} &= \sum_{n=0}^{\infty} \kappa^n a_n, \\
\frac{1}{\lambda^2} &= - \sum_{n=0}^{\infty} n \kappa^n a_n.
\end{aligned} \tag{3.11}$$

Thus, for fixed values of  $\alpha$  and  $\beta$ , since  $a'_n$ s are all known, prescribing a value of  $\kappa$  so that the sum's in (3.11) are both convergent will produce  $\lambda$  and  $s$  from the last two equations in (3.11)

as

$$\begin{aligned}\lambda &= \left( -\frac{1}{\sum_{n=0}^{\infty} n\kappa^n a_n} \right)^{1/2}, \\ s &= \lambda \sum_{n=0}^{\infty} \kappa^n a_n.\end{aligned}\tag{3.12}$$

It should be alerted that, if  $s$  is fixed in (3.11), then a numerical scheme must be employed to get  $\kappa$  and  $\lambda$ , which is not what we want here. We emphasize that expressions in (3.12) are exact formulas for the physical parameters as opposed to the numerical values existing in the literature. The valid region of  $\kappa$  may be obtained from a map by forcing the sums in (3.12) are convergent with simple exercising. This can be achieved up to the required degree of accuracy with as much terms as possible in the sum. In the more special values of  $\alpha = \beta$ , a closed-form solution can be found from (3.11) and (3.12) resulting in

$$f(\eta) = se^{\frac{\eta}{s}},\tag{3.13}$$

valid for  $s < 0$ ,  $\lambda = -\frac{1}{s}$ ,  $Q = -\frac{1}{s^2}$ , which indicates that  $Nu = -\theta'(0) = -f''(0) = -\frac{1}{s}$  is positive and hence an enhanced heat transfer rate is attained for the stream function of the form (3.13). Obviously, as the injection gets stronger, the rate of heat transfer will degrade.

### 3.2. $A_0 = f(\infty) \neq 0$ , $A_1 \neq 0$

In this case, from equation (3.8) we anticipate that

$$A_0 = \frac{\lambda^2 + Q}{\alpha\lambda},\tag{3.14}$$

and the rest of the coefficients can be found from

$$A_n = \frac{\lambda}{n(n-1)(n\lambda^2 - Q)} \sum_{k=0}^{n-1} k[(\alpha - \beta)k + \beta(2k - n)]A_k A_{n-k}, \quad n \geq 2.\tag{3.15}$$

With the help of the new scaled parameters  $Q = \lambda^2 \bar{Q}$ ,  $s = \lambda \bar{s}$ ,  $A_n = A_0 \kappa^n a_n$  with  $\kappa = \frac{A_1}{A_0}$ , (3.15) and the boundary conditions are no longer explicitly relying upon the unknown coefficients  $\lambda$  and  $A_1$ , and hence we have

$$\begin{aligned}
a_0 &= 1, \\
a_1 &= 1, \\
a_n &= \frac{1 + \bar{Q}}{\alpha n(n-1)(n-\bar{Q})} \sum_{k=0}^{n-1} k[(\alpha - \beta)k + \beta(2k - n)] a_k a_{n-k}, \quad n \geq 2, \\
\bar{s} &= \frac{1 + \bar{Q}}{\alpha} \sum_{n=0}^{\infty} \kappa^n a_n, \\
1 &= -\lambda^2 \left( \frac{1 + \bar{Q}}{\alpha} \right) \sum_{n=0}^{\infty} n \kappa^n a_n.
\end{aligned} \tag{3.16}$$

From the last two equations in (3.16),  $\lambda$  and  $\bar{s}$  can be evaluated as

$$\begin{aligned}
\lambda &= \left( -\frac{\alpha}{(1 + \bar{Q}) \sum_{n=0}^{\infty} n \kappa^n a_n} \right)^{1/2}, \\
\bar{s} &= \frac{1 + \bar{Q}}{\alpha} \sum_{n=0}^{\infty} \kappa^n a_n.
\end{aligned} \tag{3.17}$$

Therefore, for preassigned values of  $\kappa$  (and fixed  $\bar{Q}$ ,  $\alpha$  and  $\beta$ ),  $\bar{s}$  and  $\lambda$  from (3.17) can be found which are later on used to get  $s$  and  $Q$  and hence all the physical quantities are known. It is also noteworthy to draw attention that when  $\alpha = \beta$ , the exponent  $\lambda$  can be exactly worked out from (3.16) and (3.17)

$$\lambda = \frac{\alpha s \pm \sqrt{\alpha^2 s^2 + 4(\alpha - Q)}}{2}, \tag{3.18}$$

implying the existence of dual solutions for the suction case and unique solution for the injection, both depending on the heat generation/absorption parameter  $Q$  and  $\alpha$ .

## 4. Results and Discussions

We should in prior mention that our model collapse onto the well-documented one of [26], when  $\alpha = \frac{\Lambda+1}{2}$  and  $\beta = \Lambda$  in the absence of  $M$ ,  $Nr$ ,  $s$  and  $Q$ . To further validate the extracted model in the present investigation, and to justify the correctness of the exponential type solutions, Tables 1 and 2 are listed comparing the present outcomes (15 terms in the series) with those available in the open literature. Excellent agreement is anticipated in the Tables. We notice that the present model also covers the physical situation valid for the natural convection over a vertical cone formulated in [16], as inferred from Table 2.

	[27]	[6]	[7]	Present
$(\alpha = 1/2, \beta = 0.0)$	0.4437	0.4440	0.443885	0.443833
$(\alpha = 2/3, \beta = 1/3)$	0.6776	0.6788	0.677707	0.677648

Table 1: The heat transfer rates  $Nu$  at  $M = Nr = s = Q = 0$  over a vertical plate.

	[28]	[16]	Present
$(\alpha = 3/2, \beta = 0.0)$	0.7685	0.7686	0.768742
$(\alpha = 7/4, \beta = 1/2)$	0.9896	0.9897	0.989621

Table 2: The heat transfer rates  $Nu$  at  $M = Nr = s = Q = 0$  over a vertical cone.

Present solutions will be discussed next separately for  $A_0 \neq 0$  and  $A_0 = 0$ .

#### 4.1. $A_0 \neq 0$

A few of the coefficients are displayed below from (3.16)

$$\begin{aligned}
 a_0 &= 1, \\
 a_1 &= 1, \\
 a_2 &= -\frac{(\alpha - \beta)(1 + \bar{Q})}{2\alpha(-2 + \bar{Q})} \\
 a_3 &= \frac{(5\alpha - 4\beta)(\alpha - \beta)(1 + \bar{Q})^2}{12\alpha^2(-3 + \bar{Q})(-2 + \bar{Q})} \\
 a_4 &= -\frac{(\alpha - \beta)(1 + \bar{Q})^3(-68\alpha^2 + 106\alpha\beta - 42\beta^2 + (31\alpha^2 - 47\alpha\beta + 18\beta^2)\bar{Q})}{72\alpha^3(-2 + \bar{Q})^2(12 - 7\bar{Q} + \bar{Q}^2)}. \quad (4.19)
 \end{aligned}$$

When the infinity boundary condition is not vanishing, under the special circumstance  $\alpha = \beta$ , from (3.18) exponential solutions are restricted to

$$Q \leq \frac{1}{4}(4\alpha + s^2\alpha^2).$$

Moreover, there exists a critical pair  $(Q, \lambda)$  for the appearance of dual solutions which are computed as

$$(Q, \lambda) = \left( \frac{1}{4}(4\alpha + s^2\alpha^2), \frac{s\alpha}{2} \right), \quad (4.20)$$

which clearly point that in the presence of wall suction together with a positive value of  $\alpha$ , dual solutions exist as also evident from figure 2. Such solutions are likely to appear only for the heat generation case. This scenario, of course may change if negative values of  $\alpha$  are taken into account, which is also possible by the negative values of  $\Lambda$ .

Figures 3(a-c) reveal the effects of heat generation/absorption parameter  $\bar{Q}$  on the domains of  $\lambda$ ,  $\bar{s}$  and  $\kappa$  as well as on the heat transfer rate  $Nu$ , when  $\alpha = 2/3$  and  $\beta = 1/3$ . A common observation from the figures is that for  $\kappa$  larger, the exponent  $\lambda$ , the suction/injection parameter

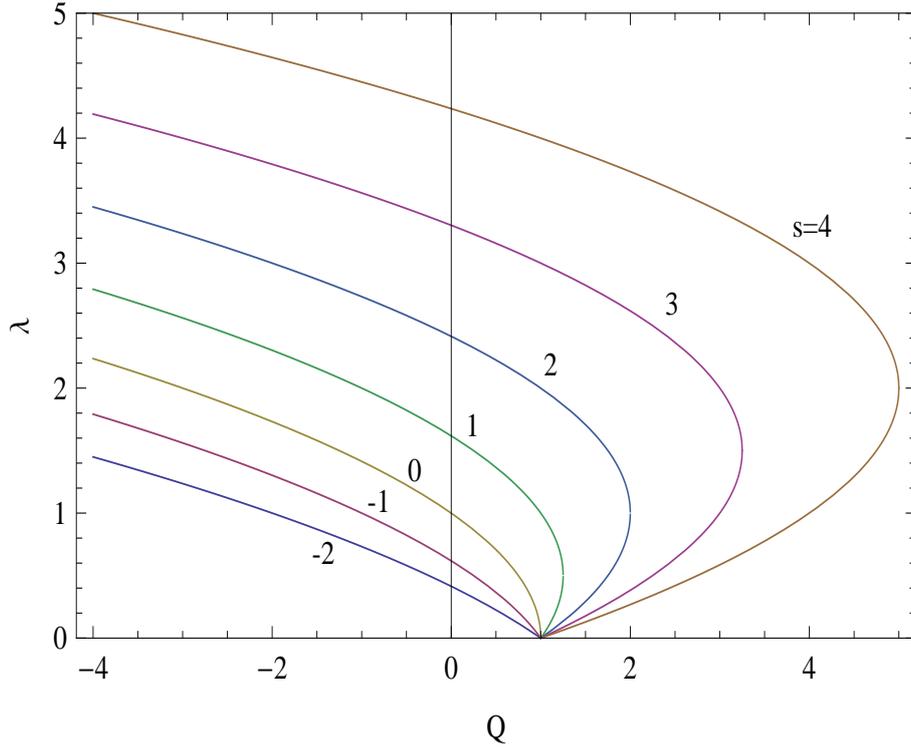


Figure 2: Domain of the existence of purely exponential solutions when  $\alpha = \beta = 1$ .

$\bar{s}$  and the Nusselt number  $Nu$  are larger, in compliance with the physical expectations. From the figures it is easy to deduce the well-known result that the suction cools down the porous medium whereas the injection heats up leading to poorer heat transfer rates. All these physical outcomes are connected with the exponent  $\lambda$ . In addition to this, absorbing the heat from the system with a negative  $\bar{Q}$  will result in better heat transfer rates as compared to the positive values of  $\bar{Q}$ , as approved from the figures. For future reference, Table 3 tabulates some values of  $\lambda$ ,  $\bar{s}$  and  $Nu$  for three values of  $\bar{Q}$  at the selected parameters  $\alpha = 2/3$ ,  $\beta = 1/3$  and  $\kappa = -1/2$ .

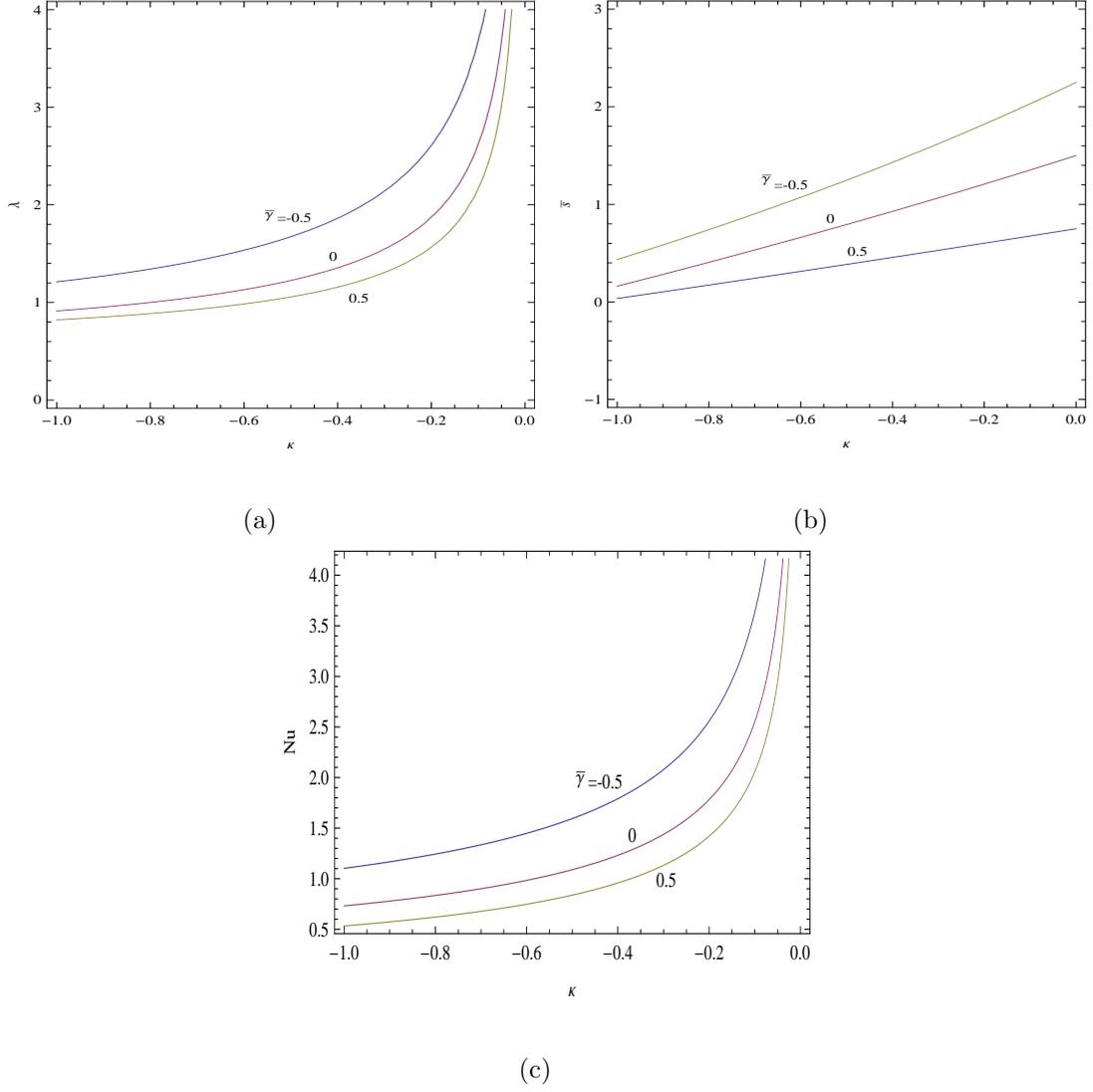


Figure 3: The effects of  $\bar{Q}$  on the physical parameters  $\lambda$ ,  $\bar{s}$ ,  $\kappa$  and Nusselt number.

$\bar{Q} = -1/2$	$\bar{Q} = 0$	$\bar{Q} = 1/2$
1.6731748403	1.2246910238	1.0563777443
0.3840527014	0.7932861455	1.2473565111
1.5940722585	1.0889155503	0.8375230026

Table 3: The values of  $\lambda$ ,  $\bar{s}$  and  $Nu$  at  $\alpha = 2/3$ ,  $\beta = 1/3$  and  $\kappa = -1/2$ .

## 4.2. $A_0 = 0$

A few of the coefficients are displayed below from (3.11)

$$\begin{aligned}
 a_0 &= 0, \\
 a_1 &= 1, \\
 a_2 &= \frac{\alpha - \beta}{6}, \\
 a_3 &= \frac{1}{144}(5\alpha - 4\beta)(\alpha - \beta), \\
 a_4 &= \frac{(\alpha - \beta)(33\alpha^2 - 51\alpha\beta + 20\beta^2)}{4320}, \\
 a_5 &= \frac{(\alpha - \beta)(443\alpha^3 - 1008\alpha^2\beta + 774\alpha\beta^2 - 200\beta^3)}{259200}, \\
 a_6 &= \frac{(\alpha - \beta)(41861\alpha^4 - 125686\alpha^3\beta + 142913\alpha^2\beta^2 - 72800\alpha\beta^3 + 14000\beta^4)}{108864000}. \quad (4.21)
 \end{aligned}$$

Figures 4(a-c) reveal the effects of  $\kappa$  on the domains of  $\lambda$ ,  $s$  as well as the on the heat transfer rate  $Nu$ , when  $\alpha = 2/3$  and  $\beta = 1/3$ . Similar behaviors to the figures 4(a-c) are exhibited. We should remark that  $Q = -\lambda^2$  in this case. For future reference, Table 4 tabulates values of  $\lambda$ ,  $s$ ,  $Q$  and  $Nu$  at the selected parameters  $\alpha = 2/3$ ,  $\beta = 1/3$  for some  $\kappa$ .

	$\lambda$	$s$	$Q$	$Nu$
$\kappa = -2$	0.7797878688	-1.4106793076	-0.6080691204	0.6450007908
$\kappa = -1$	1.0533632712	-0.9993075844	-1.1095741812	0.9507880027
$\kappa = -1/2$	1.4527010507	-0.7069776589	-2.1103403428	1.3772752129
$\kappa = -1/4$	2.0274923962	-0.4999765437	-4.1107254167	1.9730705227

Table 4: The values of  $\lambda$ ,  $s$  and  $Nu$  at  $\alpha = 2/3$ ,  $\beta = 1/3$  for some  $\kappa$ .

Finally, certain velocity and temperature profiles corresponding to Table 4 are displayed in figure 5. Figures clearly exhibit the influences of physical parameters on the flow and temper-

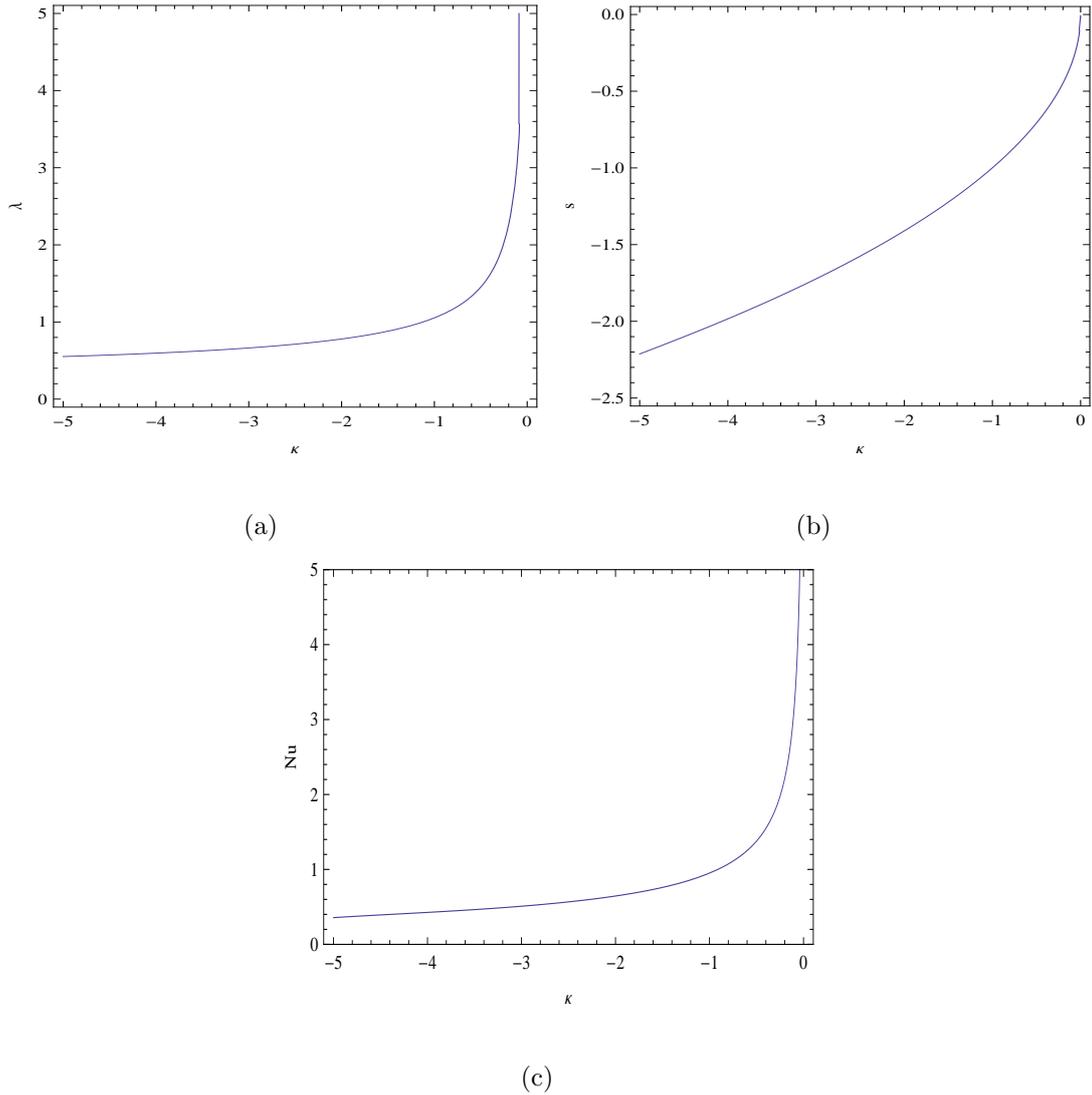


Figure 4: The effects of  $\kappa$  on the physical parameters  $\lambda$ ,  $s$  and Nusselt number.

ature fields, in parallel to the physical intuition.

## 5. Concluding remarks

The free convection flow occurring over a heated vertically stretching permeable thermally radiative magnetized surface placed in a porous medium is the main concern of the present work. The originality stems from the use of a temperature dependent internal heat generation or absorption flux, in place of the one which is preassigned in the previous studies of the

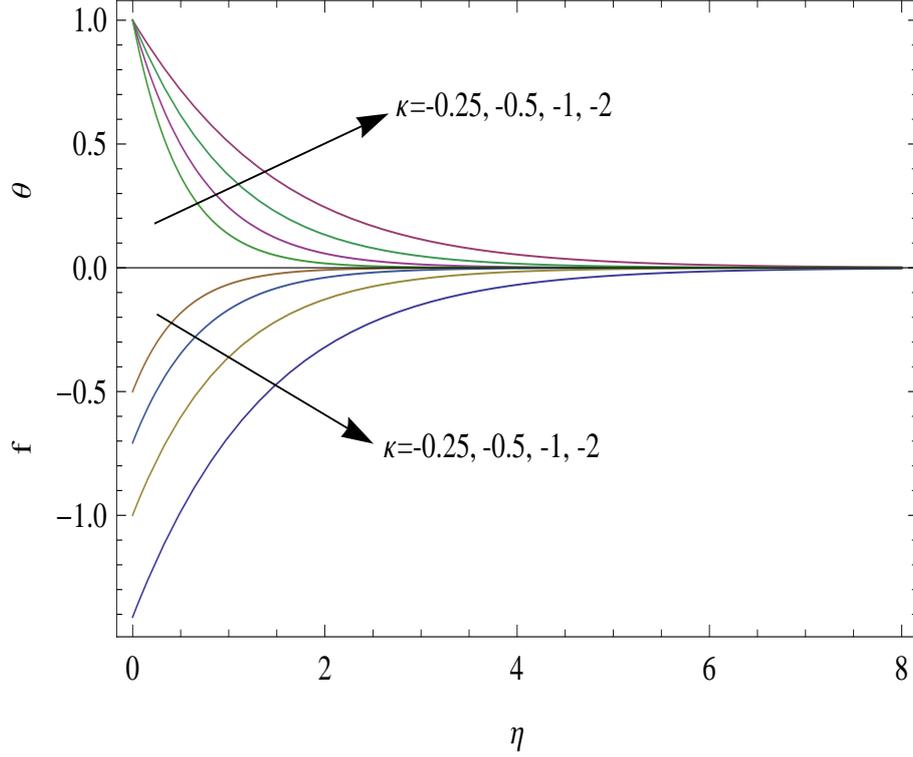


Figure 5: Velocity and temperature distributions for some prescribed parameters, see Table 4 for the corresponding parameters.

literature.

The present work is also differentiated from the literature in that the reduced nonlinear ordinary differential equation governing the physical phenomenon is treated analytically here. As a result, either closed-form exact solutions are obtained for some specific values of the physical parameters, otherwise solutions are sought in the purely exponential series form. In this case, an elegant algorithm is also proposed to determine the temperature distribution with an accuracy up to the desired decimal place without resorting to any numerical schemes.

Exact solutions are found, which are shown to be either dual or unique, depending on the governing parameters. To conclude, in the presence of a heat sink absorbing the temperature from the medium enhances the rate of heat transfer from the wall, whereas a heat source will

surely heat up the system under consideration, resulting in poorer heat transfer rates.

Since the present work generalizes the internal heat generation term suggested in [6], in future works, many publications pursuing that article may be reconsidered within context of the present model. Finally, the presented data has the potentiality to be considered as a verification tool for the natural convection processes over more complex surfaces in higher dimensions.

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