List of Modifications for

Viscoelastic Model for Bending Process of Continuous Fiber-Reinforced Thermoplastic Sheets

Reviewer A:

Comment 1: language: e.g., lines 51, 67, 322.

> **Response:** The manuscript underwent a thorough review to improve clarity. Vague sentences were rewritten throughout the text, with particular attention to lines 51, 67, and 322 (pages 3 and 7). These edits are highlighted for your reference.

Comment 2: Figures preparation: Fig. 1b) could be prepared better, cf. figures in Ch. 9 of "Composite Sheet Forming" by prof. Bhattacharyya.

> **Response:** Figure 1 illustrates two regions within the deformed geometry, along with relevant geometric parameters.

Comment 3: Choice of references: many papers dealt with the vee bending problem in different ways, see articles citing "Bending of fibre-reinforced thermoplastic sheets" by Martin, Bhatttacharyya, and Collins (1995). Setting the paper in perspective of others' research would be beneficial.

> **Response:** To enhance the literature review and provide broader context, the following references have been added.
>
> S. Ropers, M. Kardos, and T. A. Osswald, “A thermo-viscoelastic approach for the characterization and modeling of the bending behavior of thermoplastic composites,” Compos. Part A Appl. Sci. Manuf., vol. 90, pp. 22–32, 2016.
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> A. Margossian, S. Bel, and R. Hinterhoelzl, “Bending characterisation of a molten unidirectional carbon fibre reinforced thermoplastic composite using a Dynamic Mechanical Analysis system,” Compos. Part A Appl. Sci. Manuf., vol. 77, pp. 154–163, 2015.
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> P. Boisse, R. Akkerman, P. Carlone, L. Kärger, S. V Lomov, and J. A. Sherwood, “Advances in composite forming through 25 years of ESAFORM,” Int. J. Mater. Form., vol. 15, no. 3, p. 39, 2022.

N. Pyatov, H. K. Natarajan, and T. A. Osswald, “Experimental investigation of in-plane shear behaviour of thermoplastic fibre-reinforced composites under thermoforming process conditions,” J. Compos. Sci., vol. 5, no. 9, p. 248, 2021.

Comment 4: Lacking (lines 170, 258, 265) or wrongly cited references (line 153 [30] does not describe the experimental procedure) undermines the presented results.

Response: Omitted cross-references on pages 10 and 19 have been added and highlighted for your reference. An incorrect reference number on page 10 has been corrected and highlighted.

More general, comment 5: The author adopted the model of adaptive/transient links. It might be beneficial to reformulate the model in the frame of the classic Boltzmann superposition principle with relaxation functions or kernels, which is equivalent to the presented model, see discussion in the cited monograph of Drozdov. It will be beneficial both for the author and the journal as it will be more familiar to the general audience.

Response: Based on the insightful comments from the reviewer, both transient network theory and Boltzmann's superposition principle are equivalent for describing the viscoelastic material's behavior. While the transient network theory offers a more intuitive physical interpretation compared to Boltzmann's superposition principle. Additionally, the transient network framework allows for the incorporation of various hyperelastic models to capture large material deformations effectively.

More general, comment 6: Accordingly, setting the kinematics explicitly in the classic multiplicative decomposition of the deformation gradient into elastic and inelastic part ($F=F_e F_i$) would serve the same purpose.

Response: In the modified paper, the following sentences were added to explain Equation (22): ‘According to Equation (22), the deformation gradient of a link at the reformation time is the identity tensor, signifying no deformation. Subsequently, the total deformation gradient characterizes the link’s deformation after the reformation time, which depends on both the total state and the configuration at the reformation time. Within the framework of transient network theory, the links exhibit elastic behavior both before rupture and after reformation.

More general, comment 7: The claim in lines 321-325 is not supported by reference and it seems counterintuitive that lower punch velocity results in more fiber buckling.

> **Response:** The following sentence has been added to page 19 to incorporate findings from Martin et al. [1]: Winkling amplitude is greater in bending samples formed at a slower speed (50 mm/min) compared to those formed at a higher speed (500 mm/min).
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> 1. D. Bhattacharyya, Composite sheet forming. Elsevier, 1997.

Reviewer D:

1. Modeling:

Comment 1: It is not clear to Reviewer what the strain energy function w_{k} is associated with. In (6), the Author introduces the specific energy of the network, which seems to be different to the one introduced in Drozdov (1999) Acta Mechanica 133, 13-37. Following this, in line 186, the Author speaks about the "invariants of links" when referring to the formulae introduced in (4). This is confusing.

> **Response:** The concept of transient networks, introduced by Drozdov [1], has been adopted to model the strain energy within the reinforced thermoplastic sheets. This approach views the thermoplastic resin as a network of molecular chains (links) that can form and break connections. The strain energy is then expressed as the sum of the energy stored in both the initial and these adaptive links (see reference [1], equation 34).
>
> Similar to established practices in continuum mechanics, Drozdov defined the strain energy as a function of the first and second principal invariants derived from the deformation gradient tensor (see reference [1], Equation 41).
>
> To describe the application of transient networks in a continuum medium, the reference 1 was added, and the details are elaborated on pages 6-9.
>
> [1] A. D. Drozdov, "A constitutive model in finite thermoviscoelasticity based on the concept of transient networks," Acta Mech., vol. 133, no. 1, pp. 13–37, 1999.

Comment 2: I suggest the Authors comment more on how Equation (5) is obtained from reference [29].

Response: The second section has been revised to discuss the criteria for selecting a strain energy function suitable for transversely isotropic hyperelastic materials. Edits are highlighted on pages 6-9.

Comment 3: To compute the fifth invariant I_{5}, the Author makes an assumption on the direction of the fibres. However, this seems to be missing.

Response: The fibers are modeled as material lines. This allows us to determine their current direction using the deformation gradient tensor and the initial fiber direction. Edits are highlighted on page 6.

Comment 4: The Author develops the theory with a network involving K different kinds of links. Only one kind of active link is considered later on in the analysis showing good agreements. What would be the benefit/motivation of considering more types of links?

Response: The formulation was revised to explore the use of M types of links for modeling the viscoelastic response of composite sheets during bending. However, numerical results indicated that a single link type sufficiently captured the bending behavior of Plytron samples. These samples are polypropylene/glass fiber composites with a nominal fiber volume fraction of 35% and a pre-consolidated laminate stacking sequence of $[0]_8$. The general formulation, incorporating M link types, was developed to enable modeling of viscoelastic behavior in bending for a wider range of materials.

Comment 5: - In (19), W is not defined.

Response: To clarify the notation, the definition of W(t) as the total strain energy has been added on page 14 (edit highlighted).

2. Notation:

Comment 6: The dependency on time of some quantities like Y is not shown in the correct way. This should be in the same line of the text, for instance, $Y_{k}(t,\tau)$.

Response: The formulation has been revised for improved readability by presenting functions and their arguments on the same line.

3. References:

Comment 7: In some of the references an error message is printed.

Response: Omitted cross-references on pages 10 and 19 have been added and highlighted for your reference.

Comment 8: The impression of this Reviewer is that important literature on the modelling of fibre-reinforced materials is missing, e.g., the works by Ogden, Holzapfel, and Wineman, among others. Advantages and disadvantages with respect to these models will enhance the introduction.

Response: To enhance the literature review and provide broader context, the following references have been added.

S. Ropers, M. Kardos, and T. A. Osswald, "A thermo-viscoelastic approach for the characterization and modeling of the bending behavior of thermoplastic composites," Compos. Part A Appl. Sci. Manuf., vol. 90, pp. 22–32, 2016.

A. Margossian, S. Bel, and R. Hinterhoelzl, "Bending characterisation of a molten unidirectional carbon fibre reinforced thermoplastic composite using a Dynamic Mechanical Analysis system," Compos. Part A Appl. Sci. Manuf., vol. 77, pp. 154–163, 2015.

P. Boisse, R. Akkerman, P. Carlone, L. Kärger, S. V Lomov, and J. A. Sherwood, "Advances in composite forming through 25 years of ESAFORM," Int. J. Mater. Form., vol. 15, no. 3, p. 39, 2022.

N. Pyatov, H. K. Natarajan, and T. A. Osswald, "Experimental investigation of in-plane shear behaviour of thermoplastic fibre-reinforced composites under thermoforming process conditions," J. Compos. Sci., vol. 5, no. 9, p. 248, 2021.

D. Brands, L. G. di Genova, E. R. Pierik, W. J. B. Grouve, S. Wijskamp, and R. Akkerman, "Formability experiments for unidirectional thermoplastic composites," in Key Engineering Materials, 2022, vol. 926, pp. 1358–1371.

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J. Schröder and P. Neff, “Invariant formulation of hyperelastic transverse isotropy based on polyconvex free energy functions,” Int. J. Solids Struct., vol. 40, no. 2, pp. 401–445, 2003.

M. Itskov and N. Aksel, “A class of orthotropic and transversely isotropic hyperelastic constitutive models based on a polyconvex strain energy function,” Int. J. Solids Struct., vol. 41, no. 14, pp. 3833–3848, 2004.

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K. Chaimoon and P. Chindaprasirt, “An anisotropic hyperelastic model with an application to soft tissues,” Eur. J. Mech., vol. 78, p. 103845, 2019.

J. Schröder, P. Neff, and V. Ebbing, “Anisotropic polyconvex energies on the basis of crystallographic motivated structural tensors,” J. Mech. Phys. Solids, vol. 56, no. 12, pp. 3486–3506, 2008.

G. A. Holzapfel, T. C. Gasser, and R. W. Ogden, “A new constitutive framework for arterial wall mechanics and a comparative study of material models,” J. Elast. Phys. Sci. solids, vol. 61, pp. 1–48, 2000. J. Schröder, P. Neff, and D. Balzani, “A variational approach for materially stable anisotropic hyperelasticity,” Int. J. Solids Struct., vol. 42, no. 15, pp. 4352–4371, 2005.